中微习题课材料（二）

房晨

2023 年 3 月 24 日

1 Recap and Solution to PS2

1.1 Choice（最优消费选择）

- (x_1^*, x_2^*) satisfies two conditions:
  - the budget is exhausted $(p_1x_1^* + p_2x_2^* = m)$
  - The slope of the budget constraint, $-\frac{p_1}{p_2}$, and the slope of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*).

$$-\frac{\mathrm{p}_1}{\mathrm{p}_2} = \mathrm{MRS} = -\frac{\mathrm{MU}_1}{\mathrm{MU}_2}$$

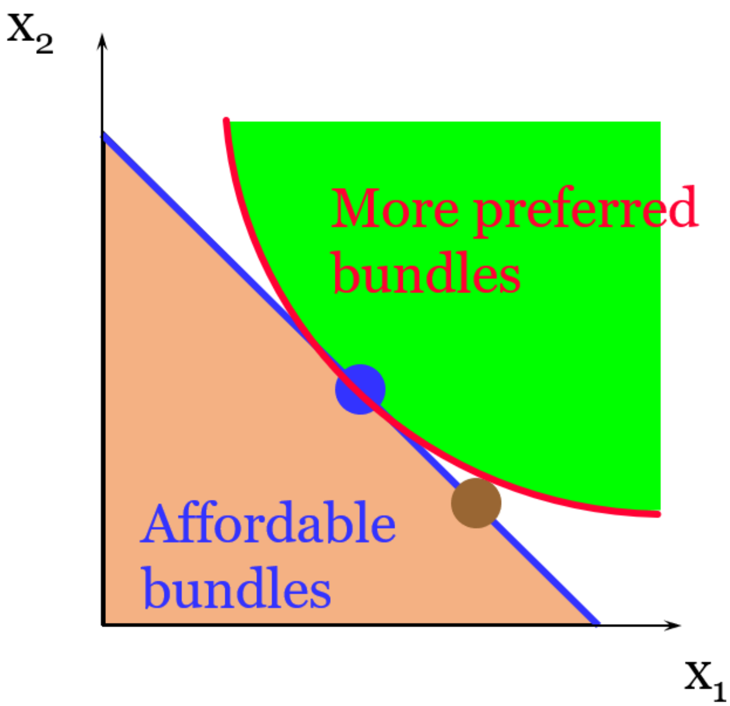


Figure 1: Rational Constrained Choice

5.7 Linus has the utility function $U(x, y) = x + 3y$.

a) On a graph below, use blue ink to draw the indifference curve passing through the point $(x, y) = (3, 3)$. Use black ink to sketch the indifference curve connecting bundles that give Linus a utility of 6.

Just as the Figure 2. The indifference curve is $U(x, y) = U(3, 3) = x + 3y = 12$ and $U(x, y) = x + 3y = 6$.

b) On the same graph, use red ink to draw Linus's budget line if the price of x is 1 and the price of y is 2 and his income is 8. What bundle does Linus choose in this situation?

(0,4) Hint: The budget line is $p_x x + p_y y = m$, which is $1 \cdot x + 2 \cdot y = 8$. We hope to maximize Linus's utility, which equals pushing the indifference curve outward as much as possible.

c) What bundle would Linus choose if the price x is 1, the price of y is 4, and his income is 8?

(8,0) Hint: The budget line is $1 \cdot x + 4 \cdot y = 8$ now. The relative price has changed, then the intersection point is on the x-axis now.

Comment: This question is about the optimal choice between perfect substitutes.

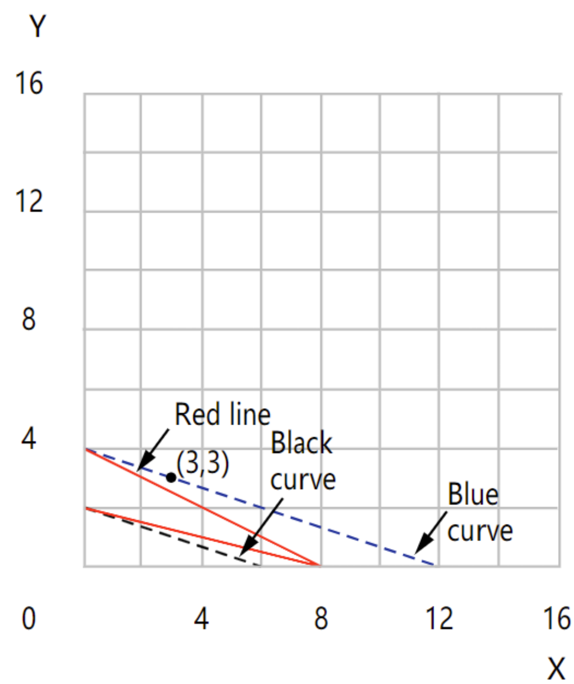


Figure 2: The graph of 5.7

5.11 *At first, refer to the 5.10 to learn the background.*

Central High School has \$60,000 to spend on computers and other stuff, so its budget equation is $C + X = 60,000$, where C is expenditure on computers and X is expenditures on other things. C.H.S. currently plans to spend \$20,000 on computers. The State Education Commission wants to encourage "computer literacy" in the high schools under its jurisdiction. The following plans have been proposed.

***Plan A:** This plan would give a grant of \$10,000 to each high school in the state that the school could spend as it wished.* $C + X = 60,000 + 10,000$

Plan B: *This plan would give a \$10,000 grant to any high school, so long as the school spent at least \$10,000 more than it currently spends on computers. Any high school can choose not to participate, in which case it does not receive the grant, but it doesn't have to increase its expenditure on computers.* *when* $C \geq 30,000$, $C + X = 60,000 + 10,000$

Plan C: *Plan C is a "matching grant" (配套给予) For every dollar's worth of computers that a high school orders, the state will give the school* 50 *cents.* $C + X = 60,000 + 0.5C$

Plan D: *This plan is like plan C, except that the maximum amount of matching funds that any high school could get from the state would be limited to \$10,000.* *when* $C \leq 20,000$, $C + X = 60,000 + 0.5C$

Suppose that Central High School has preferences that can be represented by the utility function $U(C, X) = CX^2$. Let us try to determine how the various plans described in the last problem will affect the amount that C.H.S. spends on computers.

(a) If the state adopts none of the new plans, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.

20,000. Hint: Using the conclusion of Rational Constrained Choice: $-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$. The price of C and X can be both considered as 1.

(b) If plan A is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.

23,333. Hint: The budget constrain line becomes $C + X = 70,000$ while other things being the same.

(c) On your graph, sketch the indifference curve that passes through the point $(30,000, 40,000)$ if plan B is adopted. At this point, which is steeper, the indifference curve or the budget line?

The budget line. Hint: The slope of the budget line is -1, while the slope of the indifference curve at the point $(30,000, 40,000)$ is $-\frac{MU_1}{MU_2} = -\frac{X}{2C} = -\frac{2}{3}$, whose absolute value is less than 1.

(d) If plan B is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. (Hint: Look at your graph.)

30,000. Hint: You cannot move the indifference curve further from the graph Figure 3.

(e) If plan C is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.

40,000. Hint:$-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$. The budget constrain line of Plan C is $0.5C + X = 60,000$ while $-\frac{p_1}{p_2}$ changes to be $-\frac{0.5}{1} = -0.5$.

(f) If plan D is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.

23,333. Hints: Please be clear about the position of the point of tangency (on which part of the budget line).

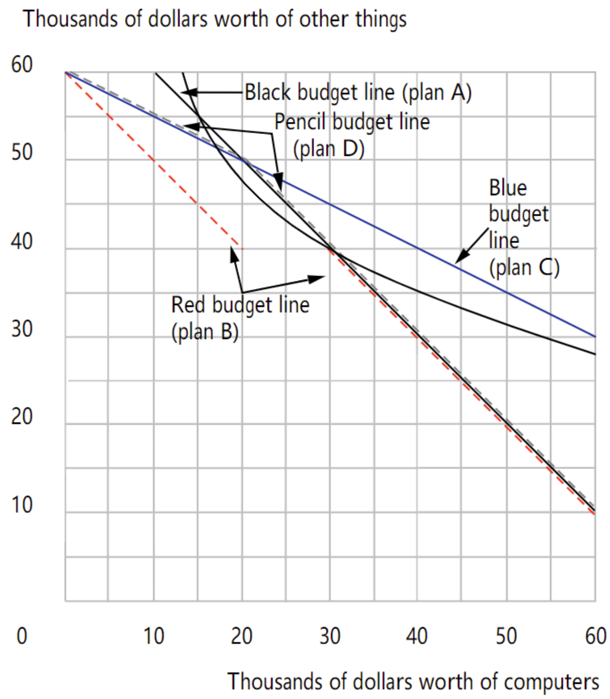


Figure 3: The graph of 5.11

1.2 Individual Demand（个体需求）

- Own-Price Changes
  - Price offer curve
  - Ordinary demand curve (and Inverse demand)
  - Giffen v.s. Ordinary Goods
- Income Changes
  - Income offer curve
  - Engel curve
  - Homotheticity and Quasilinear Utility
  - Inferior v.s. Normal Goods

6.7 Mary's utility function is $U(b,c) = b + 100c - c^2$, where b is the number of silver bells (银铃) in her garden and c is the number of cockle shells (麦仙翁). She has 500 square feet in her garden to allocate between silver bells and cockle shells. Silver bells each take up 1 square foot and cockle shells each take up 4 square feet. She gets both kinds of seeds for free.

(a) To maximize her utility, given the size of her garden, Mary should plant 308 silver bells and 48 cockle shells. (Hint: Write down her "budget constraint" for space. Solve the problem as if it were an ordinary demand problem.)

Budget constrain is: $b + 4c = 500$. Solve it using $-\frac{p_1}{p_2} = -\frac{1}{4} = MRS = -\frac{MU_1}{MU_2} = -\frac{1}{100-2c}$. $c = 48$ and $b = 308$.

(b) If she suddenly acquires an extra 100 square feet for her garden, how much should she increase her planting of silver bells? 100 extra silver bells. How much should she increase her planting of cockle shells? Not at all.

Budget constrain is: $b + 4c = 500 + 100 = 600$. Solve it using $-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$ again. $c = 48$ and $b = 408$.

(c) If Mary had only 144 square feet in her garden, how many cockle shells would she grow? 36.

Budget constrain is: $b + 4c = 144$. The solution from $-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$ doesn't hold here. Only corner solution. (Just as the Figure 4)

(d) If Mary grows both silver bells and cockle shells, then we know that the number of square feet in her garden must be greater than 192. Hint: At least pass the point of $(48, 0)$.

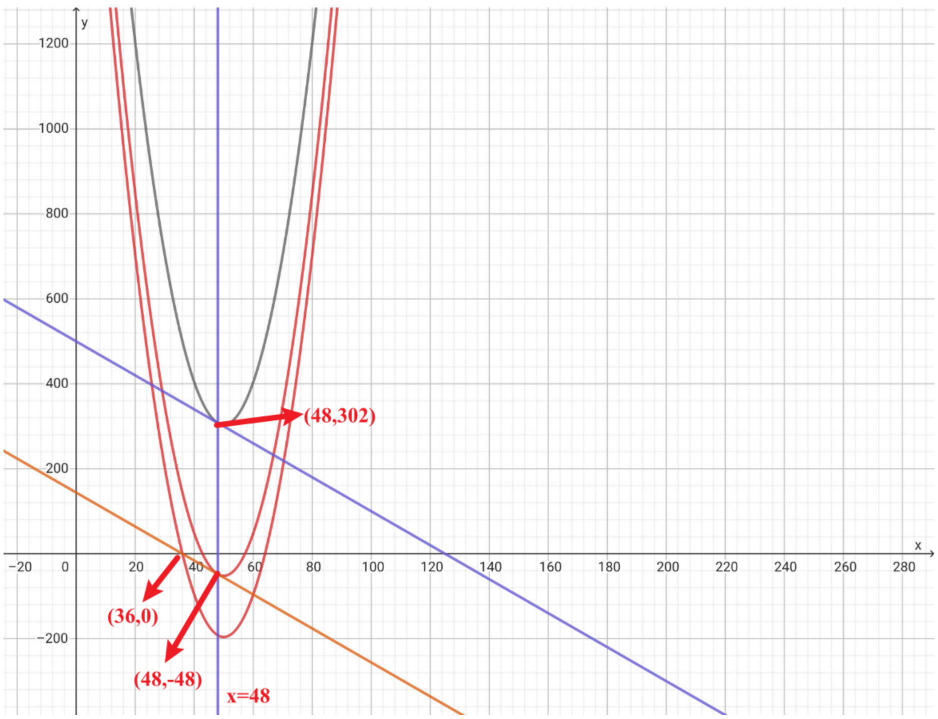


Figure 4: The graph of 6.7

Comment: These indifference curves are similar to Quasilinear Utility.

6.12 As you may remember, Nancy Lerner is taking an economics course in which her overall score is the minimum of the number of correct answers she gets on two examinations. ($S(T_1, T_2) = min(T_1, T_2)$)For the first exam, each correct answer costs Nancy 10 minutes of study time. For the second exam, each correct answer costs her 20 minutes of study time. In the last chapter, you found the best way for her to allocate 1200 minutes between the two exams. Some people in Nancy's class learn faster and some learn slower than Nancy. Some people will choose to study more than she does, and some will choose to study less than she does. In this section, we will find a general solution for a person's choice of study times and exam scores as a function of the time costs of improving one's score.

(a) Suppose that if a student does not study for an examination, he or she gets no correct answers. Every answer that the student gets right on the first examination costs P_1 minutes of studying for the first exam. Every answer that he or she gets right on the second examination costs P_2 minutes of studying for the second exam. Suppose that this student spends a total of M minutes studying for the two exams and allocates the time between the two exams in the most efficient possible way. Will the student have the same number of correct answers on both exams? Yes. (If the numbers of correct answers in both exams are not the same, the student will have a motivation to adjust his/her study time to increase the lower number while slightly decrease the higher number until the two numbers equal to each other. $T_1 = T_2$) Write a general formula for this student's overall score for the course as a function of the three variables, P_1, P_2 and M: $S = \frac{M}{P_1+P_2}$ ($P_1T_1 + P_2T_2 = M$ and $T_1 = T_2$). If this student wants to get an overall score of S, with the smallest possible total amount of studying, this student must spend P_1S minutes studying for the first exam and P_2S studying for the second exam.

(b) Suppose that a student has the utility function $U(S, M) = S - \frac{A}{2}M^2$, where S is the student's overall score for the course, M is the number of minutes the student spends studying, and A is a variable that reflects how much the student dislikes studying. In Part (a) of this problem, you found that a student who studies for M minutes and allocates this time wisely between the two exams will get an overall score of $S = \frac{M}{P_1+P_2}$. Substitute $\frac{M}{P_1+P_2}$ for S in the utility function and then differentiate with respect to M to find the amount of study time, M, that maximizes the student's utility. $M = \frac{1}{A(P_1+P_2)}$ ($U(M) = \frac{M}{P_1+P_2} - \frac{A}{2}M^2$ and use the first order condition). Your answer will be a function of the variables P_1, P_2, and A. If the student chooses the utility-maximizing amount of study time and allocates it wisely between the two exams, he or she will have an overall score for the course of $S = \frac{1}{A(P_1+P_2)^2}$ (Because $S = \frac{M}{P_1+P_2}$).

(c) Nancy Lerner has a utility function like the one presented above. She chose the utility-maximizing amount of study time for herself. For Nancy, $P_1 = 10$ and $P_2 = 20$. She spent a total of $M = 1,200$ minutes studying for the two exams. This gives us enough information to solve for the variable A in Nancy's utility function. In fact, for Nancy, $A = \frac{1}{36,000}$ (Because $M = \frac{1}{A(P_1+P_2)}$).

(d) Ed Fungus is a student in Nancy's class. Ed's utility function is just like Nancy's, with the same value of A. ($U(S, M) = S - \frac{1}{2*36,000}M^2$) But Ed learns more slowly than Nancy. In fact it takes Ed exactly twice as long to learn anything as it takes Nancy, so that for him, $P_1 = 20$ and $P_2 = 40$. Ed also chooses his amount of study time so as to maximize his utility. Find the ratio of the amount of time Ed spends studying to the amount of time Nancy spends studying. $\frac{1}{2}$ (Because $M = \frac{1}{A(P_1+P_2)}$, the studing time of Ed is 600 miutes). Will his score for the course be greater than half, equal to half, or less than half of Nancy's? Less than half. (Because $S = \frac{1}{A(P_1+P_2)^2}$, Nancy's score is 40 while Ed's score is only 10.)

EQ3 Consider Logan, a consumer who has preferences represented by the utility function $U(H, J) = H^{\frac{2}{3}} + J^{\frac{2}{3}}$, where H represents the number of healthy meals Logan consumes per week and J represents the number of unhealthy (junk) meals that Logan consumes per week. Logan has an income of \$42 per week. The price of healthy meals and unhealthy meals are each \$2 per meal. Find Logan's utility-maximizing consumption bundle. What proportion of the meals that Logan consumes are healthy?

Now suppose that Logan's parents offer to pay for 50% of Logan's healthy meals, thereby lowering the price of a healthy meal to \$1. Find Logan's new utility-maximizing consumption bundle. Now what proportion of the meals that Logan consumes are healthy?

Solution: First, we find Logan's marginal rate of substitution:

$$\frac{dJ}{dH} = -\frac{\frac{\partial U}{\partial H}}{\frac{\partial U}{\partial J}} = -\frac{\frac{2}{3}H^{-\frac{1}{3}}}{\frac{2}{3}J^{-\frac{1}{3}}} = -\left(\frac{J}{H}\right)^{\frac{1}{3}}.$$

Her marginal rate of substitution is decreasing, because as we move to the right on an indifference curve, H increases, and this causes her marginal rate of substitution to decrease (H is on the bottom of the fraction). This means that her preferences are strictly convex. For her to be maximizing her utility the following two conditions must be met: (1) her marginal rate of substitution is equal to the negative of the price ratio, and (2) she must spend all of her income, that is she chooses a bundle on her budget line.

Setting her MRS equal to the negative of the price ratio we obtain

$$\begin{aligned}
&-\left(\frac{J}{H}\right)^{\frac{1}{3}} = -\frac{2}{2} \\
&\Leftrightarrow \left(\frac{J}{H}\right)^{\frac{1}{3}} = 1 \\
&\Leftrightarrow \left(\frac{J}{H}\right)^{\frac{3}{3}} = 1^3 \\
&\Leftrightarrow \frac{J}{H} = 1 \\
&\Leftrightarrow J = H
\end{aligned}$$

The equation of her budget line is $2H + 2J = 42$.

Plugging $J = H$ into her budget line equation we obtain

$$\begin{aligned}
&2H + 2H = 42 \\
\Rightarrow &4H = 42 \\
\Rightarrow &H = \frac{42}{4} = 10.5
\end{aligned}$$

From $J = H$, we then get that her utility-maximizing bundle is $(H, J) = (10.5, 10.5)$. Half of Logan's meals are healthy.

For the second part of the problem, the price ratio is now $\frac{p_H}{p_J} = \frac{1}{2}$. Setting marginal rate of substitution equal to the negative of the price ratio, we obtain:

$$\begin{aligned}
&-\left(\frac{J}{H}\right)^{\frac{1}{3}} = -\frac{1}{2} \\
&\Leftrightarrow \left(\frac{J}{H}\right)^{\frac{3}{3}} = \left(\frac{1}{2}\right)^3 \\
&\Leftrightarrow \frac{J}{H} = \frac{1}{8} \\
&\Leftrightarrow J = \frac{H}{8} \\
&\Leftrightarrow H = 8J
\end{aligned}$$

The equation of her budget line is now $H + 2J = 42$.

Plugging $H = 8J$ into her budget line equation we obtain

$$\begin{aligned} 8J + 2J &= 42 \\ \Rightarrow 10J &= 42 \\ \Rightarrow \quad J &= \frac{42}{10} = 4.2 \end{aligned}$$

Plugging $J = 4.2$ into $H = 8J$, we get that $H = 33.6$. The utility-maximizing bundle is now $(H, J) = (33.6, 4.2)$. Healthy meals now account for $\frac{33.6}{33.6+4.2} = 88.88\%$ of Logan's diet.

1.3 Revealed Preference（显示偏好）

- Direct and Indirect Preference Revelation
- The Weak Axiom of Revealed Preference (WARP): If $X \succ_D Y$, then never is $Y \succ_D X$.
- The Strong Axiom of Revealed Preference (SARP): $X \succ_D Y$ or $X \succ_I Y$, not $Y \succ_D X$ and $Y \succ_I X$.
- Quantity Index Numbers

  Laspeyres quantity index:

$$\mathrm{L_q} = \frac{\mathrm{p_1^b x_1^t + p_2^b x_2^t}}{\mathrm{p_1^b x_1^b + p_2^b x_2^b}}$$

  Paasche quantity index:

$$\mathrm{P_q} = \frac{\mathrm{p_1^t x_1^t + p_2^t x_2^t}}{\mathrm{p_1^t x_1^b + p_2^t x_2^b}}$$

- Price Index Numbers

  Laspeyres price index:

$$\mathrm{L_p} = \frac{\mathrm{p_1^t x_1^b + p_2^t x_2^b}}{\mathrm{p_1^b x_1^b + p_2^b x_2^b}}$$

  Paasche price index:

$$\mathrm{P_p} = \frac{\mathrm{p_1^t x_1^t + p_2^t x_2^t}}{\mathrm{p_1^b x_1^t + p_2^b x_2^t}}$$

7.9 The McCawber family is having a tough time making ends meet. They spend \$100 a week on food and \$50 on other things. A new welfare program has been introduced that gives them a choice between receiving a grant of \$50 per week that they can spend any way they want, and buying any number of \$2 food coupons (食品优惠券) for \$1 a piece. (They naturally are not allowed to resell these coupons.) Food is a normal good for the McCawbers. As a family friend, you have been asked to help them decide on which option to choose. Drawing on your growing fund of economic knowledge, you proceed as follows.

(a) On the graph below, draw their old budget line in red ink and label their current choice C. Now use black ink to draw the budget line that they would have with the grant. ($Otherthings + Food = \$150 + \$50$) If they chose the coupon option, how much food could they buy if they spent all their money on food coupons? $300. (Using all of their income to buy the coupons, $\$150 \times 2 = \$300$) How much could they spend on other things if they bought no food? $150. Use blue ink to draw their budget line if they choose the coupon option. ($Otherthings + 0.5 * Food = \$150$)

(b) Using the fact that food is a normal good for the McCawbers, and knowing what they purchased before, darken the portion of the black budget line where their consumption bundle could possibly be if they chose the lump-sum grant option. Label the ends of this line segment A and B. (The food is a normal good, it means if the purchase power/income increases, they will buy more food to satisfy themselves, more than $100.)

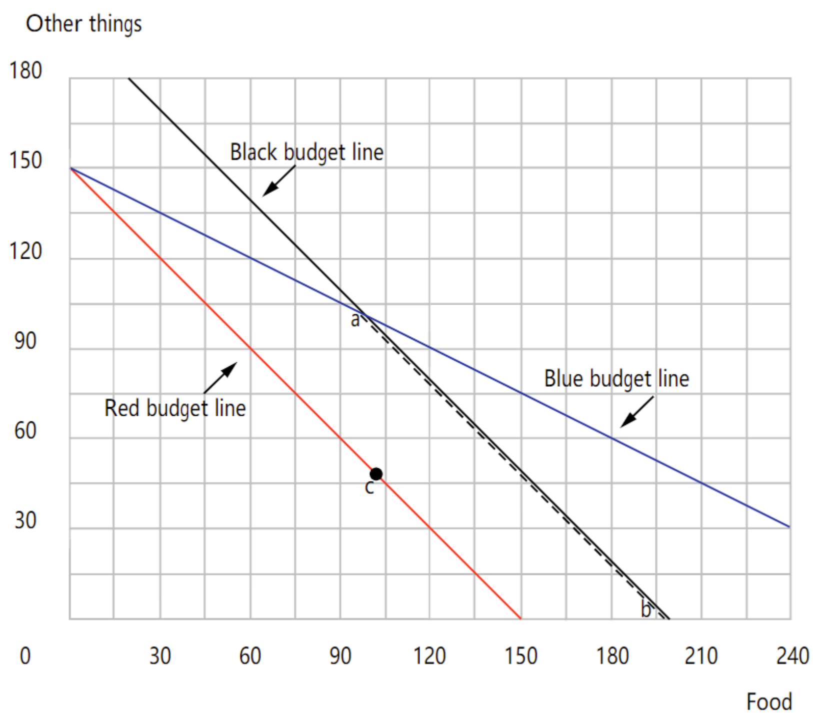


Figure 5: The graph of 7.9 (a) & (b)

(c) After studying the graph you have drawn, you report to the McCawbers. "I have enough information to be able to tell you which choice to make. You should choose the coupon because you can get more food even when other expenditure is constant."

(d) Mr. McCawber thanks you for your help and then asks, "Would you have been able to tell me what to do if you hadn't known whether food was a normal good for us?" On the axes below, draw the same budget lines you drew on the diagram above, but draw indifference curves for which food is not a normal good and for which the McCawbers would be better off with the program you advised them not to take. If the food is the inferior good, then the number consumed by McCawber will decrease if they have the access to the grant or coupons from the government.

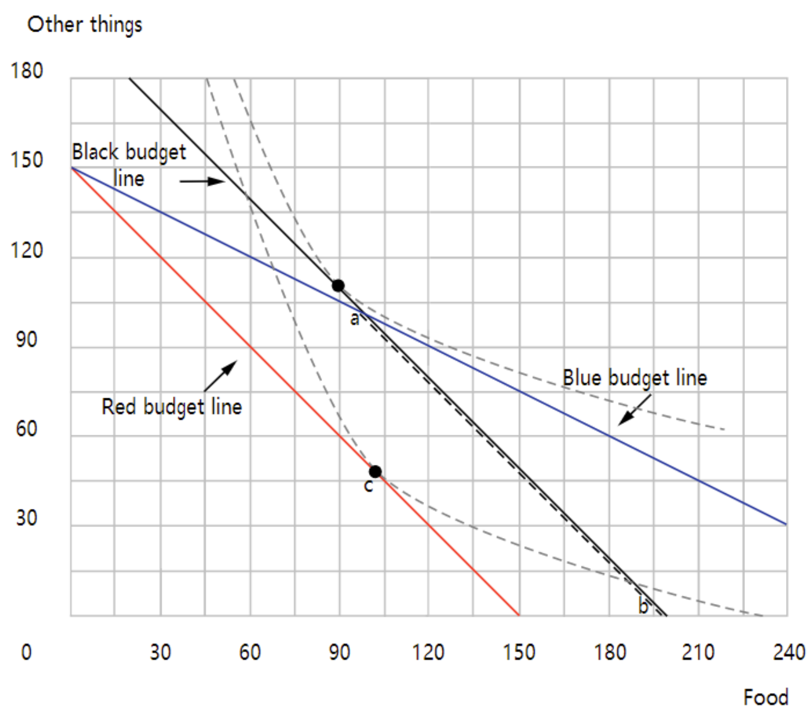


Figure 6: The graph of 7.9 (d)

EQ4 There are three commodities available for consumption. In week 1, the prices of these commodities are $(p_1, p_2, p_3) = (4, 2, 2)$, and you observe Antwan consume quantities $(x_1, x_2, x_3) = (6, 3, 2)$. In week 2, the prices of the commodities are $(p_1, p_2, p_3) = (3, 4, 1)$, and you observe Antwan consume quantities $(x_1, x_2, x_3) = (2, 4, 6)$. In week 3, the prices of the commodities are $(p_1, p_2, p_3) = (1, 5, 2)$, and you observe Antwan consume quantities $(x_1, x_2, x_3) = (4, 2, 8)$. You may assume that Antwan has monotonic preferences (单调偏好). Show that Antwan's choices satisfy the Weak Axiom of Revealed Preference (WARP). Do Antwan's choices satisfy the Strong Axiom of Revealed Preference (SARP)? Assuming that Antwan's choices do not change, is there an alternative price of commodity 1 in week 1, such that his choices violate WARP? Please explain your answers.

Solution: For simplicity of exposition, label bundle $(6, 3, 2)$ as A; $(2, 4, 6)$ as B; and $(4, 2, 8)$ as C. In the table below, the number in the cell is the cost of that column's bundle at that row's price. For example, bundle C costs \$36 at week 1 prices.

		Choices		
		A: (6,3,2)	B: (2,4,6)	C: (4,2,8)
Prices	Week 1: (4,2,2)	34	28	36
	Week 2: (3,4,1)	32	28	28
	Week 3: (1,5,2)	25	34	30

(1) *Weak Axiom of Revealed Preference (WARP)*: If $X \succ_D Y$, then never is $Y \succ_D X$.

Week 1: B is affordable (costs 28 –income is 34), but A is chosen. Thus, $A \succ_D B$.

Week 2: C is affordable (costs 28 –income is 28), but B is chosen. Thus, $B \succ_D C$.

Week 3: A is affordable (costs 25 –income is 30), but C is chosen. Thus, $C \succ_D A$.

Thus, there are no violations of *WARP*.

(2) *SARP*: $X \succ_D Y$ or $X \succ_I Y$, not $Y \succ_D X$ and $Y \succ_I X$.

From (1):

$$A \succ_D B \text{ and } B \succ_D C \Rightarrow A \succ_I C$$
$$B \succ_D C \text{ and } C \succ_D A \Rightarrow B \succ_I A$$
$$C \succ_D A \text{ and } A \succ_D B \Rightarrow C \succ_I B$$

SARP violations:

$$A \succ_I C \text{ and } C \succ_D A;$$
$$B \succ_I A \text{ and } A \succ_D B;$$
$$C \succ_I B \text{ and } B \succ_D C.$$

(3) Alternative p_1 in Week 1 such that Antwan's choices violate *WARP*:

From Week 3, $C \succ_D A$. Therefore, if C is affordable in Week 1 (when A was chosen), we'd have $A \succ_D C$, which would violate *WARP*. Hence, we need

$$\begin{aligned}&\text{Cost A} \geq \text{ Cost C}\\ \Rightarrow &6p_1 + 3(2) + 2(2) \geq 4p_1 + 2(2) + 8(2)\\ \Rightarrow &p_1 \geq 5.\end{aligned}$$

1.4 Slutsky Decomposition（斯勒茨基分解）

- total effect of a price change
  - relative price change: substitution effect (替代效应)
  - purchasing power change: income effect (收入效应)

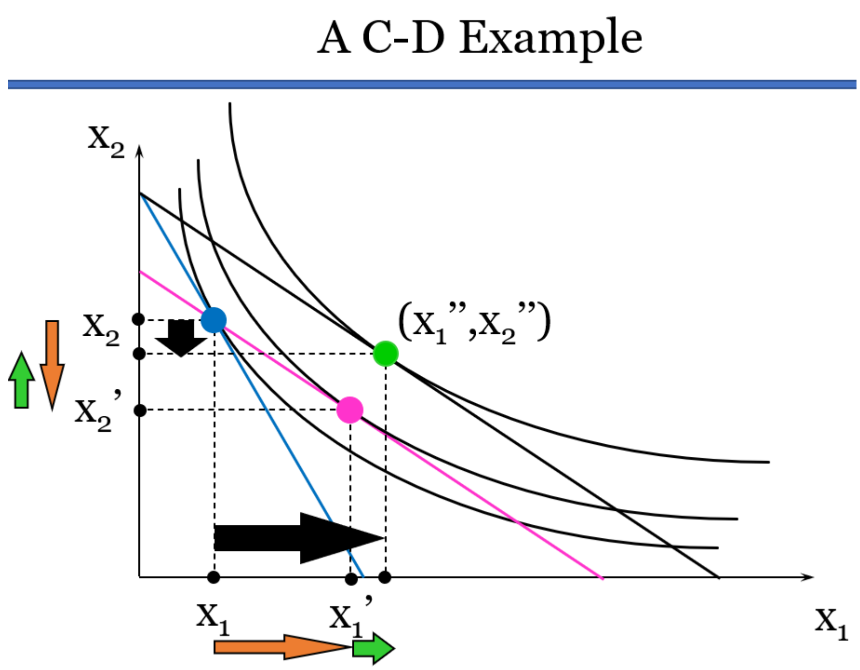


Figure 7: A C-D Example of Slutsky Equation

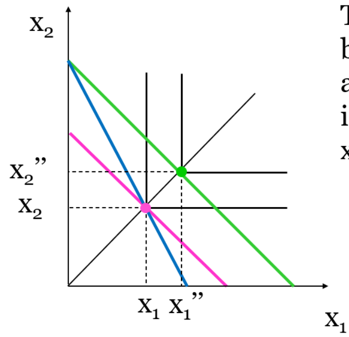
$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$
$$\Delta x_1^s = x_1(p_1', p_2,\ m') - x_1(p_1, p_2,\ m)$$
$$\Delta x_1^n = x_1(p_1', p_2,\ m) - x_1(p_1', p_2,\ m')$$

- **The Law of Demand**: If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases. (i.e. normal goods must have downward-sloping demand curves)
- ...

8.5 Suppose that two goods are perfect complements. If the price of one good changes, what part of the change in demand is due to the substitution effect, and what part is due to the income effect?

All income effect.

Perfect complements



The "intermediate" bundle is the same as the old bundle.
i.e. $x_1 = x_1', x_2 = x_2'$

Perfect complements

The "intermediate" bundle is the same as the old bundle.
i.e. $x_1 = x_1', x_2 = x_2'$

$$x_1^s = x_1' - x_1 = 0$$

- no substitution effect

$$x_1^n = x_1'' - x_1' = x_1'' - x_1$$

- Total effect = Income effect

Figure 8: The graph of 8.5

EQ5 Ambrose's utility function for goods 1 and 2 is $u(x_1, x_2) = 4\sqrt{x_1} + x_2$. Suppose Ambrose has \$48 to spend on goods 1 and 2, that the price of good 2 is $P_2 = 8$, and that the price of good 1 falls from $P_1 = 4$ to $P_1 = 2$. Calculate Ambrose's original optimal bundle, his new optimal bundle after the price decrease, and the income and substitution effects on good 1. Show your work.

Now suppose that Ambrose has \$256 to spend on goods 1 and 2, that the price of good 2 is $P_2 = 8$, and that the price of good 1 falls from $P_1 = 4$ to $P_1 = 2$. Again, calculate Ambrose's original optimal bundle, his new optimal bundle after the price decrease, and the income and substitution effects on good 1. Show your work.

Solution:

$$MRS = -\frac{2}{\sqrt{x_1}}$$

Tangency between indifference curves and the budget line requires

$$\frac{2}{\sqrt{x_1}} \triangleq \frac{P_1}{P_2}, \text{ or } x_1 = \frac{4P_2^2}{P_1^2}.$$

When $P_1 = 4$, $P_2 = 8$, tangency occurs at $x_1 = 16$, which costs 64.

When he has only \$48 to spend, his indifference curves are always steeper than budget line. Therefore, it requires a corner solution with $x_2 = 0$, $x_1 = 12$.

When $P_1 = 2$, $P_2 = 8$, optimality still requires a corner solution with $x_2'' = 0$, $x_1'' = 24$.

The hypothetical budget line reflects the new price ratio, has slope $-\frac{P_1}{P_2} = -\frac{1}{4}$, but goes through the old optimum $(x_1', x_2') = (12, 0)$. An equation for the budget line is

$$2x_1 + 8x_2 = 24.$$

The optimum along the hypothetical budget line is $(x_1', x_2') = (12, 0)$. For the drop in P_1, there is **no substitution effect**, only **an income effect of 12** on good 1 consumption.

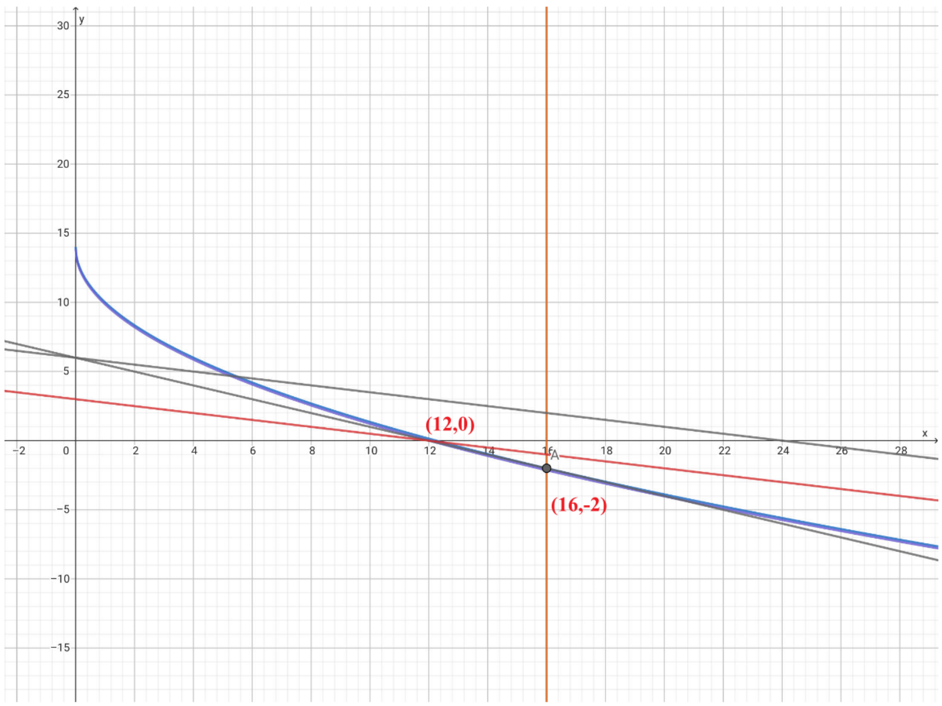


Figure 9: The graph of EQ5_1

When he has \$256 to spend and $P_1 = 4$, $P_2 = 8$, he can afford $x_1 = 16$, then $x_2 = 24$.

If the price of good 1 falls to $P_1 = 2$, then he can afford the tangency condition $x_1'' = 64$, then $x_2'' = 16$.

The hypothetical budget line goes through the point (16,24), but reflects the new prices $P_1 = 2$, $P_2 = 8$. An equation for this hypothetical budget line is

$$2x_1 + 8x_2 = 224.$$

The hypothetical optimum along this hypothetical budget line is $x_1' = 64$, then $x_2' = 12$.

When the price of good 1 fell, the substitution effect is an increase of 48 in good 1 consumption, and a decrease of 12 in good 2 consumption. The income effect is an increase of 4 in good 2 consumption (and no income effect on good 1 due to the decrease in P_1).

In general, if good 1 is the good that enters the utility function in a non-linear way, and if the consumer has enough money to achieve tangency, changes in P_1 only results in substitution effects on good 1 consumption, not income effect.

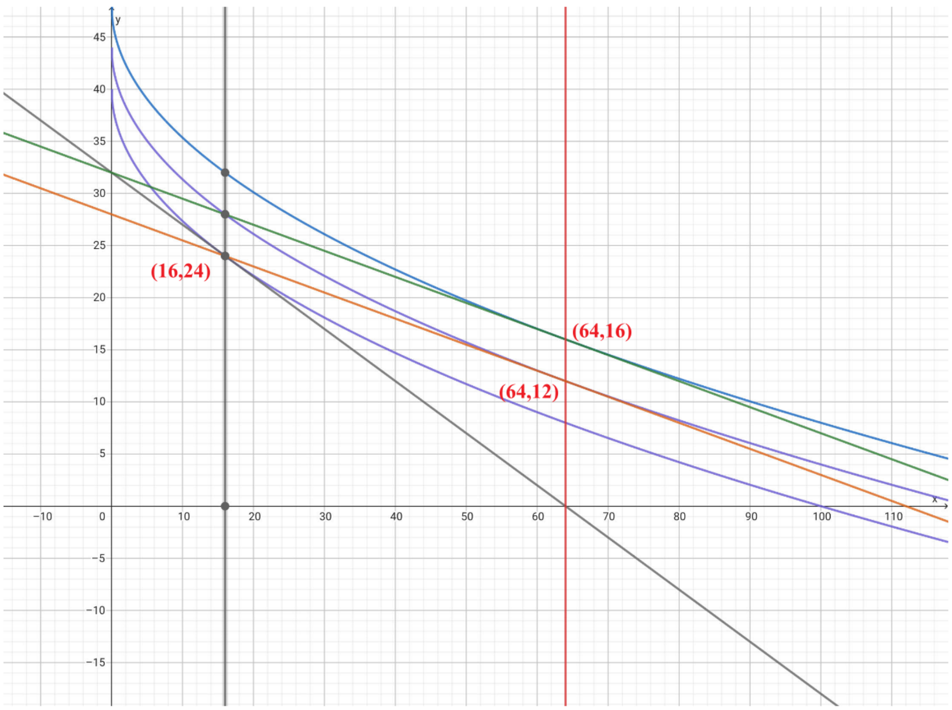


Figure 10: The graph of EQ5_2

Comment: The corner solution and quasi-linear utility.

2 Supplement Materials

2.1 Constrained Optimization: Method of Lagrange Multipliers

The consumer's optimization problem:

$$\begin{aligned} & \max_{x_1,x_2\geq 0} u\left(x_1,x_2\right) \\ \text{s.t.}\quad & p_1x_1+p_2x_2\leq m \end{aligned}$$

The Lagrange function (拉格朗日函数) is

$$\mathcal{L}=u\left(x_1,x_2\right)+\lambda\left(m-p_1x_1-p_2x_2\right)$$

where $\mathcal{L}$ is the Lagrange multiplier (拉格朗日乘数).

If (x_1^*,x_2^*) is the optimal choice, it must satisfy

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial x_1} &= \frac{\partial u\left(x_1^*,x_2^*\right)}{\partial x_1}-\lambda p_1=0 \\ \frac{\partial\mathcal{L}}{\partial x_2} &= \frac{\partial u\left(x_1^*,x_2^*\right)}{\partial x_2}-\lambda p_2=0 \\ \frac{\partial\mathcal{L}}{\partial \lambda} &= p_1x_1^*+p_2x_2^*-m=0 \end{aligned}$$

Rearranging yields:

$$\begin{aligned} \frac{\partial u\left(x_1^*,x_2^*\right)}{\partial x_1} &= \lambda p_1 \\ \frac{\partial u\left(x_1^*,x_2^*\right)}{\partial x_2} &= \lambda p_2 \end{aligned}$$

Combining these two equations, we get the familiar first order condition

$$\text{MRS}_{12}\left(x_1^*,x_2^*\right)=\frac{p_1}{p_2}$$

One obvious advantage of the method of Lagrange multipliers is that it is general enough to deal with choices with more than two goods.

Consider

$$\begin{aligned} & \max_{x_l\geq 0} u\left(x_1,\cdots,x_n\right) \\ \text{s.t.}\quad & \textstyle\sum_{l=1} p_lx_l\leq m \end{aligned}$$

If $(x_1^*,...,x_n^*)$ is the optimal choice, then it satisfies

$$\frac{\partial\mathcal{L}}{\partial x_i}=\frac{\partial u\left(x_1^*,\ldots,x_n^*\right)}{\partial x_i}-\lambda p_i=0$$

λ equals the "shadow price" of the budget constraint, i.e. it expresses the quantity of utilities that could be obtained with the next dollar of consumption.

Example 1

Maximize $u = 4x^2 + 3xy + 6y^2$, subject to $x + y = 56$.

Example 2: Maximizing Consumer Utility with Three Goods

A consumer consumes three goods, X, Y and Z. Her utility function is $U(X, Y, Z) = X^2YZ$. Her income is \$10, and price of each good is \$1.

Example 3: Cost Minimization

A firm produces two goods, x and y. Due to a government quota, the firm must produce subject to the constraint $x + y = 42$. The firm's cost functions is

$$c(x, y) = 8x^2 - xy + 12y^2$$

Reference Links:

Economic Applications of Lagrange Multipliers

Optimization with Constraints The Lagrange Multiplier Method

The Lagrange Method (Lecture Notes of Intermediate Microeconomics)

2.2 Additional Questions

1 **Utility:** The absolute value of Mars' MRS at his current consumption bundle is greater than 3. (That is, $\frac{MU_1}{MU_2} > 3$). Mars has convex preferences and is currently consuming positive amounts of both goods.

(a) Taking away some of Good 1 and giving Mars 3 units of Good 2 for each unit of Good 1 taken away will necessarily make him worse off.

(b) Taking away some Good 1 and giving Mars 3 units of Good 2 for each unit of Good 1 taken away will necessarily make him better off.

(c) Giving Mars some Good 1 and taking away 3 units of Good 2 for each unit of Good 1 he is given will necessarily make him worse off.

(d) Giving Mars some Good 1 and taking away 3 units of Good 2 for each unit of Good 1 he is given will necessarily make him better off.

(e) More than one of the above is true.

2 **Choice:** Justin consumes goods X and Y and has a utility function $U(x, y) = x^2 + y$. The price per unit of X is p_x and the price per unit of Y is p_y. He has enough money so that he can afford at least 1 unit of either good. When he chooses his best affordable bundle, it must necessarily be that:

(a) his budget line is tangent to the indifference curve passing through this bundle.

(b) he consumes only x.

(c) he consumes only y if p_x^2/p_y exceeds his income.

(d) he consumes some of each good if $p_x = p_y$.

(e) he consumes some of each good if $p_y = \frac{p_x}{2}$.

3 **Demand:** Fred consumes pork chops and lamb chops and nothing else. When the price of pork chops rises with no change in his income or in the price of lamb chops. Fred buys fewer lamb chops and fewer pork chops. From this information we can defnitely conclude that

(a) pork chops are a normal good for Fred.

(b) lamb chops are a normal good for Fred.

(c) pork chops are an inferior good for Fred.

(d) lamb chops are an inferior good for Fred.

(e) Fred prefers pork chops to lamb chops.

4 **Revealed Preference:** When the prices were $(3, 1)$, Zelda chose the bundle $(x, y) = (8, 7)$. Nw at the new prices (p_x, p_y), she chooses the bundle $(x, y) = (7, 9)$. For Zelda's behavior to be consistent with the weak axiom of revealed preference, it must be that:

(a) $2p_y < p_x$.

(b) $p_x < 2p_y$.

(c) $3p_y < p_x$.

(d) $p_y = 3p_x$.

(e) None of the above.

5 **Slutsky Equation:** Cindy consumes goods x and y. Her demand for a is given by $x(p_x, m) = 0.05m - 5.25p_x$. Now her income is 545. the price of x is 4. and the price of y is 1. If the price of a rises to 5 and if we denote the income effect on her demand for x by DI and the substitution effect on her demand for x by DS, then:

(a) $DI = -0.31$ and $DS = -0.52$.

(b) $DI = -0.31$ and $DS = -4.94$.

(c) $DI = -0.52$ and $DS = -0.52$.

(d) $DI = 0$ and $DS = -2.00$.

(e) None of the above.

6 Nadia likes pork Ribs (R) and Chicken wings (C). Her utility function is $U(R, C) = 10R^2C$. Her weekly income is \$90 which she spends exclusively on R and C. The price for a slab of ribs is \$10 and \$5 for a piece chicken. (Answer parts a to f).

(a) State in words and in math Nadia's consumer problem.

Solution: Nadia want to choose the bundle (R, C) that maximizes her utility subject to her budget constraint.

$$\text{Max}_{R,C} \quad U(R, C) = 10R^2C$$
$$\text{s.t.} \;\; 10R + 5C \leq 90$$

(b) What is Nadia's optimal bundle?

Solution: As the U-function is of the Cobb-Douglas type we know that the Indifference Curves are "nice and convex". Hence, the optimum bundle satisfies the slope condition and is also on the budget line.

Slope condition:

$$\frac{MU_R}{MU_C} = \frac{P_R}{P_C} \to \frac{2C}{R} = 2 \to R = C$$

Budget line:

$$90 = 10R + 5C \to 10R + 5R = 90 \to 15R = 90 \to R^* = 6, C^* = 6$$

(c) What is her demand function for ribs?

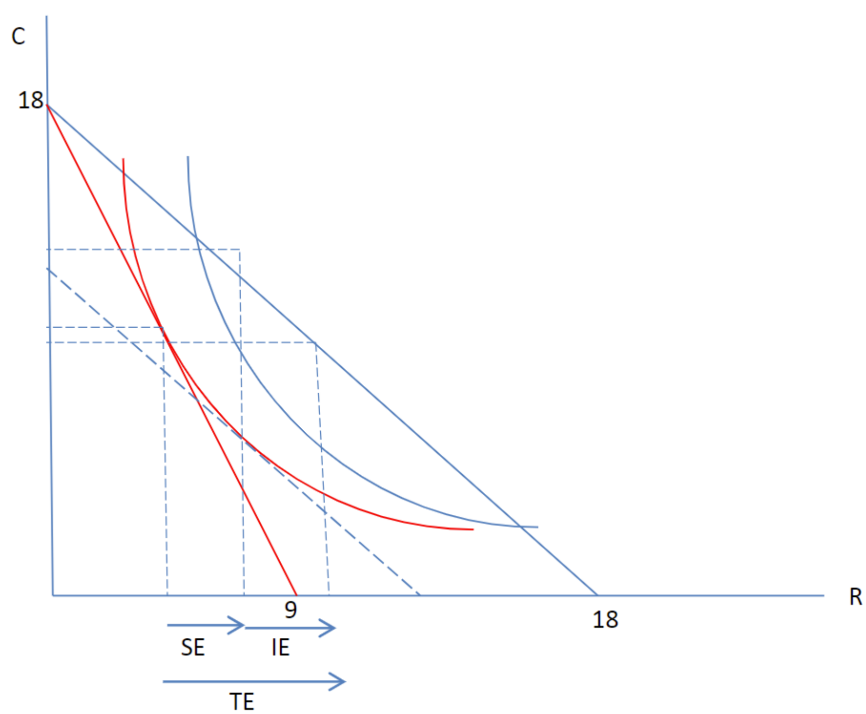
Solution:

$$\frac{2C}{R} = \frac{P_R}{5} \to C = \frac{P_R R}{10}$$
$$90 = P_R R + 5\left(\frac{P_R R}{10}\right) \to P_R R + \frac{1}{2} P_R R = 90 \to \frac{3}{2} P_R R = 90 \to R(P_R) = \frac{60}{P_R}$$

(d) Are ribs a normal or inferior good?

Solution: Normal (because demand increases as income increases).

(e) The price of ribs falls to \$5. Draw the income and substitution effects of this price change graphically. Assume ribs are on the horizontal axis.



(f) What would Nadia's optimal bundle be if her utility function was given by $U = \sqrt{R} + \sqrt{C}$? Assume the price for a slab of ribs is \$10 and \$5 for chicken.

Solution:

$$MRS = \frac{\frac{1}{2}R^{-\frac{1}{2}}}{\frac{1}{2}C^{-(\frac{1}{2})}} = \left(\frac{C}{R}\right)^{\frac{1}{2}} = 2 \rightarrow \frac{C}{R} = 4 \rightarrow C = 4R$$

$$90 = 10R + 5(4R) \rightarrow 90 = 30R \rightarrow R^* = 3, C^* = 12$$

7 Dynamic Consumption and Saving (20 pts) [1]

Suppose that a village plants crops in $t = 0$ and the crops will produce output in $t = 1, 2, 3$. Output is decreasing over time where $R_1 = 4$, $R_2 = 1$, $R_3 = 0$. The village only consumes in periods $t = 1, 2, 3$, so the world ends in $t = 3$.

The village can store crop output across time periods **without any depreciation or uncertainty**. There is **no discounting of future periods**. The utility function from consumption is $u(C_t) = \sqrt{C_t}$. If the village stores S_t crops in period $t-1$, then it can consume up to S_t crops plus any output R_t in period t, but minus any storage for the next period $S_t + 1$.

(a) (6 pts) Set up the village's maximization problem to find the optimal consumption and storage plan. Be clear about the objective function and constraints.

Solution:

$$\begin{aligned} \max_{C_t, S_t} \quad & \textstyle\sum_{t=1}^{3} \sqrt{C_t} \\ \text{s.t.} \quad & C_1 = R_1 - S_2 \\ & C_2 = R_2 + S_2 - S_3 \\ & C_3 = R_3 + S_3 \end{aligned}$$

(b) (4 pts) Notice that the utility function is strictly concave and there is no discounting over time. Do you have any conjectures about what the optimal consumption plan looks like?

Solution: Since utility is strictly concave, we do not want extreme amounts of consumption in each period. Since there is no discounting, we value consumption in each time period equally, and therefore utility is maximized by setting consumption equal across time.

(c) (6 pts) Write the FOCs for the optimal consumption and storage amounts C_t for $t = 1, 2, 3$ and S_t for $t = 2, 3$.

Solution: We can rewrite the maximization problem by subbing in R_1, R_2, plugging C_t into the objective so we have a function in S_2, S_3

$$\max \sqrt{4 - S_2} + \sqrt{1 + S_2 - S_3} + \sqrt{S_3}$$

leading to the FOCs:

$$-\frac{1}{2}(4 - S_2)^{-1/2} + \frac{1}{2}(1 + S_2 - S_3)^{-1/2} = 0$$

$$-\frac{1}{2}(1 + S_2 - S_3)^{-1/2} + \frac{1}{2}(S_3)^{-1/2} = 0$$

(d) (4 pts) Solve for optimal consumption and storage C_t, S_t.

[1] This question is from an Intermediate Microeconomics Midterm Exam.

Solution: Rearranging the FOCs, we find that $S_2 = \frac{7}{3}$, and $S_3 = \frac{5}{3}$. We can plug these back into the consumption constraints to get $C_1 = C_2 = C_3 = \frac{5}{3}$. Clearly, we can see that it is optimal to smooth consumption over time since the utility function is concave and there is no discounting.