中微习题课材料（四）

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1 Recap and Solution to PS4

1.1 Market Demand（市场需求）

- To get market demand, just add up individual demands.
  - add horizontally
- Often think of market behaving like a single individual.
  - representative consumer model.
  - not true in general, but reasonable assumption for this course.
- Elasticity（弹性）: the "sensitivity" of one variable with respect to another.
  - The elasticity of variable X with respect to variable Y is

$$\epsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}$$

  - measures responsiveness of demand to price

$$\epsilon = \frac{p}{q}\frac{dq}{dp}$$

  - own-price elasticity of demand（自身价格弹性）and cross-price elasticity of demand（交叉价格弹性）
  - point elasticity（点弹性）and arc-elasticity（弧弹性）
  - example for linear demand curve
    - for linear demand, $q = a - bp$, so $\epsilon = -bp/q = -bp/(a - bp)$
    - note that $\epsilon = -1$ when we are halfway down the demand curve
  - suppose demand takes form $q = Ap^{-b}$

$$\epsilon = -\frac{p}{q}bAp^{-b-1} = \frac{-bAp^{-b}}{Ap^{-b}} = -b$$

    - thus elasticity is constant along this demand curve
    - note that: $log(q) = log(A) - b * log(p)$

  - What does elasticity depend on? In general how many and how close substitutes a good has.

- How does revenue change when you change price?
  - $R = pq$, so $\Delta R = (p + dp)(q + dq) - pq = pdq + qdp + dpdq$
  - last term is very small relative to others.
  - $dR/dp = q + pdq/dp$
  - $dR/dp > 0$ when $|\epsilon| < 1$

- How does revenue change as you change quantity?
  - marginal revenue: $MR = dR/dq = p + qdp/dq = p[1 + 1/\epsilon]$
  - **elastic**: absolute value of elasticity greater than 1
  - **inelastic**: absolute value of elasticity less than 1
  - Monopolist never sets a price where $|\epsilon| < 1$ —because it could always make more money by reducing output.

- Marginal revenue curve
  - always the case that $dR/dq = p + qdp/dq$.
  - in case of linear (inverse) demand, $p = a - bq$, $MR = dR/dq = p - bq = (a - bq) - bq = a - 2bq$.

15.5 The demand function for drangles is $q(p) = (p + 1)^{-2}$.

(a) What is the price elasticity of demand at price p?

$-2p/(p + 1)$

(b) At what price is the price elasticity of demand for drangles equal to -1?

When the price equals 1.

(c) Write an expression for total revenue from the sale of drangles as a function of their price. $R(p) = pq = p/(p + 1)^2$ Use calculus to find the revenue-maximizing price. Don't forget to check the second-order condition. Differentiating and solving gives $p = 1$.

(d) Suppose that the demand function for drangles takes the more general form $q(p) = (p + a)^{-b}$ where $a > 0$ and $b > 1$. Calculate an expression for the price elasticity of demand at price p. $-bp/(p + a)$. At what price is the price elasticity of demand equal to -1? $p = a/(b - 1)$.

15.10 The athletic director discussed in the last problem is considering the extra revenue he would gain from three proposals to expand the size of the football stadium. Recall that the demand function he is now facing is given by $q(p) = 300,000 - 10,000p$.

(a) How much could the athletic director increase the total revenue per game from ticket sales if he added $1,000$ new seats to the stadium's capacity and adjusted the ticket price to maximize his revenue?

$9,900$.

(b) How much could he increase the revenue per game by adding $50,000$ new seats? $\$250,000$. $60,000$ new seats? (Hint: The athletic director still wants to maximize revenue.) $\$250,000$.

(c) A zealous alumnus offers to build as large a stadium as the athletic director would like and donate it to the university. There is only one hitch. The athletic director must price his tickets so as to keep the stadium full. If the athletic director wants to maximize his revenue from ticket sales, how large a stadium should he choose?

150,000 seats.

1.2 Equilibrium（均衡）

- Equilibrium
  - competitive market —each agent takes prices as outside his or her control
    - many small agents
    - a few agents who think that the others keep fixed prices
  - equilibrium price —that price where desired demand equals desired supply
    - $D(p) = S(p)$
  - special cases
    - vertical supply —quantity determined by supply, price determined by demand
    - horizontal supply —quantity determined by demand, price determined by supply
  - an equivalent definition of equilibrium: where inverse demand curve crosses inverse supply curve
    - $P_d(q) = P_s(q)$
  - examples with linear curves
- Taxes —nice example of comparative statics
  - demand price and supply price —different in case of taxes
  - $p_d = p_s + t$
  - equilibrium happens when $D(p_d) = S(p_s)$
  - put equations together:
    - $D(p_s + t) = S(p_s)$
    - or $D(p_d) = S(p_d - t)$
  - also can solve using inverse demands:
    - $P_d(q) = P_s(q) + t$
    - or $P_d(q) - t = P_s(q)$
- Deadweight loss of a tax
  - benefits to consumers
  - benefits to producers
  - value of lost output

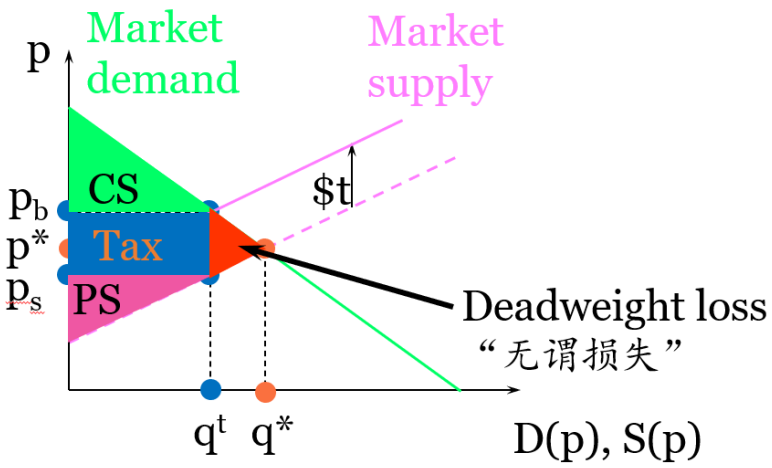

Figure 1: The graph of Deadweight Loss

16.6 Schrecklich and LaMerde are two justifiably obscure nineteenth century impressionist painters. The world's total stock of paintings by Schrecklich is 100, and the world's stock of paintings by LaMerde is 150. The two painters are regarded by connoisseurs (鉴赏家) as being very similar in style. Therefore the demand for either painter's work depends both on its own price and the price of the other painter's work. The demand function for Schrecklichs is $D_S(P) = 200-4P_S-2P_L$, and the demand function for LaMerdes is $D_L(P) = 200-3P_L-P_S$, where P_S and P_L are respectively the price in dollars of a Schrecklich painting and a LaMerde painting.

(a) Write down two simultaneous equations that state the equilibrium condition that the demand for each painter's work equals supply.

The equations are $200-4P_S-2P_L = 100$ and $200-3P_L-P_S = 150$.

(b) Solving these two equations, one finds that the equilibrium price of Schrecklichs is 20 and the equilibrium price of LaMerdes is 10.

(c) On the diagram below, draw a line that represents all combinations of prices for Schrecklichs and LaMerdes such that the supply of Schrecklichs equals the demand for Schrecklichs. Draw a second line that represents those price combinations at which the demand for LaMerdes equals the supply of LaMerdes. Label the unique price combination at which both markets clear with the letter E.

(d) A fire in a bowling alley in Hamtramck, Michigan, destroyed one of the world's largest collections of works by Schrecklich. The fire destroyed a total of 10 Schrecklichs. After the fire, the equilibrium price of Schrecklichs was 23 and the equilibrium price of LaMerdes was 9.

(e) On the diagram you drew above, use red ink to draw a line that shows the locus of price combinations at which the demand for Schrecklichs equals the supply of Schrecklichs after the fire. On your diagram, label the new equilibrium combination of prices E .

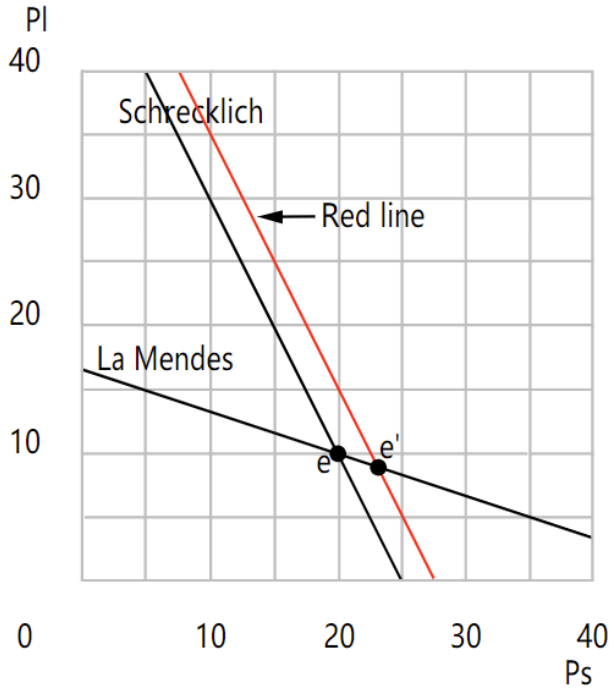

Figure 2: The graph of 16.6

16.11 King Kanuta rules a small tropical island, Nutting Atoll, whose primary crop is coconuts. If the price of coconuts is P, then King Kanuta's subjects will demand $D(P) = 1,200-100P$ coconuts per week for their own use. The number of coconuts that will be supplied per week by the island's coconut growers is $S(P) = 100P$

(a) The equilibrium price of coconuts will be 6 and the equilibrium quantity supplied will be 600.

(b) One day, King Kanuta decided to tax his subjects in order to collect coconuts for the Royal Larder. The king required that every subject who consumed a coconut would have to pay a coconut to the king as a tax. Thus, if a subject wanted 5 coconuts for himself, he would have to purchase 10 coconuts and give 5 to the king. When the price that is received by the sellers is p_S, how much does it cost one of the king's subjects to get an extra coconut for himself? $2p_S$.

(c) When the price paid to suppliers is p_S, how many coconuts will the king's subjects demand for their own consumption? (Hint: Express p_D in terms of p_S and substitute into the demand function.) Since $p_D = 2p_S$, they consume $1,200-200p_S$.

(d) Since the king consumes a coconut for every coconut consumed by the subjects, the total amount demanded by the king and his subjects is twice the amount demanded by the subjects. Therefore, when the price received by suppliers is p_S, the total number of coconuts demanded per week by Kanuta and his subjects is $2,400-400p_S$.

(e) Solve for the equilibrium value of p_S 24/5 , the equilibrium total number of coconuts produced 480 , and the equilibrium total number of coconuts consumed by Kanuta's subjects. 240.

(f) King Kanuta's subjects resented paying the extra coconuts to the king, and whispers of revolution spread through the palace. Worried by the hostile atmosphere, the king changed the coconut tax. Now, the shopkeepers who sold the coconuts would be responsible for paying the tax. For every coconut sold to a consumer, the shopkeeper would have to pay one coconut to the

king. This plan resulted in $480/2 = 240$ coconuts being sold to the consumers. The shopkeepers got $24/5$ per coconut after paying their tax to the king, and the consumers paid a price of $48/5$ per coconut.

EQ7 The number of bottles of artisanal maple syrup（手工枫糖浆）demanded per year is $D(p) = 180 - p^2$. The number of bottles supplied is $S(p) = 3p$. Find the equilibrium price and quantity. What is the elasticity of demand in this equilibrium?

Suppose that the government introduces a new subsidy such that in addition to the price paid by consumers, the government pays suppliers $\$x$ per bottle sold. Find the new supply function for maple syrup, and calculate the equilibrium price of syrup as a function of the subsidy, x. If the amount of the subsidy is \$1.80 per bottle, what is the price paid by consumers and the price received by the suppliers (rounded to the nearest cent)? Please show your work and explain your answers.

Solution: The demand for syrup is $D(p) = 180 - p^2$, and the supply of syrup is $S(p) = 3p$.

To determine the equilibrium price and quantity, we set $D(p) = S(p)$. This gives

$$\begin{aligned} & p^2 + 3p - 180 = 0 \\ \Rightarrow & (x - 12)(x + 15) = 0 \\ \Rightarrow & p^* = 12. \end{aligned}$$

Plugging this into either the supply or the demand function gives us $D(12) = S(12) = Q^* = 36$.

The equation for elasticity of demand is $\epsilon = \frac{dD}{dp}\frac{p}{D}$. Taking the derivative of the demand function, we get $\frac{dD}{dp} = -2p$. Plugging the equilibrium price and quantity into the elasticity equation gives

$$\epsilon = (-2)(12)(\frac{12}{36}) = -8$$

When the government introduces the subsidy of $\$x$ per bottle to the firms, the firms receive the price paid by consumers plus the amount of the subsidy. That is, $p^s = p + x$, where $p^D = p$. The firm's new supply function is $S(p^s) = 3(p + x)$

In equilibrium,

$$\begin{aligned} & D(p) = S(p + x) \\ \Rightarrow & 180 - p^2 = 3(p + x) \\ \Rightarrow & p^2 + 3p - 180 + 3x = 0 \\ \Rightarrow & p^* = \left(\frac{1}{2}\right)\sqrt{729 - 12x} - 1.5. \end{aligned}$$

If the amount of the subsidy is $x = \$1.80$, then the price paid by consumers is $p^D = p = 11.80$, and the price received by suppliers is $p^S = p + 1.80 = 13.60$.

2 Additional Questions

1 **Elasticity:** The constant elasticity of demand for cigarettes has been estimated to be 0.5. To reduce smoking by 75%, approximately how much tax needs to be added to a $1 pack?

(a) $0.38

(b) $0.75

(c) $1.50

(d) $2.25

(e) $4.00

2 **Market Demand:** If the demand function is $q = m - 2ln(p)$ over some range of values of p, then at all such values of p the absolute value of the price elasticity of demand:

(a) increases as p increases.

(b) decreases as p increases.

(c) is constant as p changes.

(d) increases with p at small values and decreases with p at large values

(e) decreases with p at large values and increases with p at small values

3 **Equilibrium:** The market for tennis shoes has a horizontal supply curve and a linear, downward-sloping demand curve. Currently the government imposes a tax of t on every pair of tennis shoes sold and does not tax other goods. The government is considering a plan to double the tax on tennis shoes, while leaving other goods untaxed. If the tax is doubled, then

(a) the total deadweight loss caused by the doubled tax will be exactly twice the original deadweight loss.

(b) the total deadweight loss caused by the doubled tax will be more than twice the original deadweight loss.

(c) the total deadweight loss caused by the doubled tax will be less than twice the original deadweight loss.

(d) to know if doubling the tax would more than double the deadweight loss, we would have to know theslope of the demand curve.

(e) None of the above.

4 **Equilibrium:** Remember King Kanuta and his tropical island? The demand function for coconuts by his subjects on the island is $D(p) = 1200 - 100p$ and the supply function is $S(p) = 100p$. The law used to be that any subject who consumed a coconut had to pay another coconut to the king. King Kanuta then ate all the coconuts he got. But now the king, apparently fed up with coconuts, decides to sell the coconuts that he collects in the local market at the going selling price, p_s. In equilibrium the number of coconuts that will now be produced is

(a) 100.

(b) 200.

(c) 600.

(d) 400.

(e) 300.

5 The inverse demand function for bananas is $P_d = 18 - 3Q_d$ and the inverse supply function is $P_s = 6 + Q_s$, where prices are measured in cents.

(a) If there are no taxes or subsidies, what is the equilibrium quantity? 3. What is the equilibrium market price? 9 cents.

(b) If a subsidy of 2 cents per pound is paid to banana growers, then in equilibrium it still must be that the quantity demanded equals the quantity supplied, but now the price received by sellers is 2 cents higher than the price paid by consumers. What is the new equilibrium quantity? 3.5. What is the new equilibrium price received by suppliers? 9.5 cents. What is the new equilibrium price paid by demanders? 7.5 cents.

(c) Express the change in price as a percentage of the original price. -16.66%. If the cross-elasticity of demand between bananas and apples is +.5, what will happen to the quantity of apples demanded as a consequence of the banana subsidy, if the price of apples stays constant? (State your answer in terms of percentage change.) -8.33%.

3 Comment on Midterm Exam

- 因为阅卷时是踩点给分，所以强烈建议答题时写**关键点**，不宜长篇大论而重点不突出
- **书写**还是很重要，尽量整洁、清晰，也减少因为写得不清楚而漏判的失分
- 计算题会给**步骤**分，所以步骤要尽量清晰，否则若只有答案而答案又不正确，就非常吃亏
- 对于课件上的基本**概念**（如拉氏价格指数）、**推导**（如斯勒茨基分解）、**例题**要多熟悉