The Geography of Development

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Outline

Outline

Motivation

Why do we need to understand the Geography of Development?

- How do **migration restrictions** affect the evolution of the world economy?
- How do they interact with production centers to shape the economy of the future?

Driven Forces and Elements

- Unique relative to other locations $\Rightarrow$ costs of trading, amenities, productivity
- Migration across/within countries $\Rightarrow$ possible but limited, restrictions or frictions
- **Dynamic** labor productivity $\Rightarrow$ institutions, infrastructure, education, capital stocks
- Population density $\Rightarrow$ innovation, agglomeration effects and costs of congestion

What do the researchers do?

Structure

- Allen and Arkolakis (2014): **static** spatial equilibrium
- Eaton and Kortum (2002): migration, local factors, trade structure
- Kline and Moretti (2014): heterogeneous preferences
- Desmet and Rossi-Hansberg (2014): **dynamic** version (invest in improving local technology, explicitly modeled)
- Zabreyko et al. (1975): uniqueness of the equilibrium, steady state, initial distribution, simulation

Contributions

- Identify local characteristics: land prices $\Rightarrow$ income per capita + population counts (Desmet and Rossi-Hansberg, 2013) (Allen and Arkolakis, 2014) (Fajgelbaum and Redding, 2014) (Behrens et al., 2017)
- Free mobility $\Rightarrow$ incorporate migration **frictions** within/across countries
- Subjective well-being data $\Rightarrow$ information on the **welfare** of individuals (Deaton, 2008) (Kahneman and Deaton, 2010)
- ...

Preview of Findings

- Relaxing migration restrictions leads to large increases in **output** and **welfare** at impact
- One of key determinants is the correlation between GDP per capita and population density
- **Venezuela, Brazil, & Mexico**: become world's **densest** and most **productive** countries

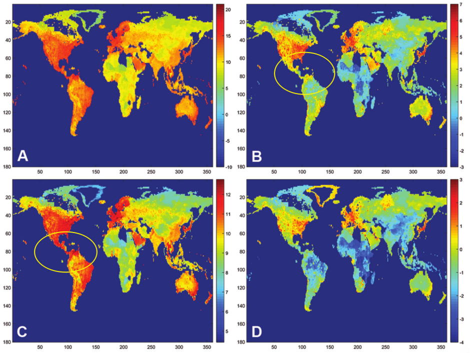

Figure: Equilibrium with free migration (period 1). **A** Population density; **B** Productivity; **C** Utility; **D** Real income per capita

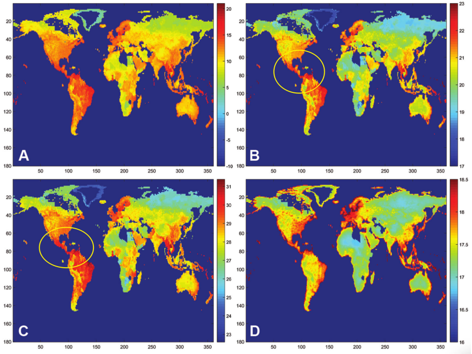

Figure: Equilibrium with free migration (period 600). **A** Population density; **B** Productivity; **C** Utility; **D** Real income per capita

Outline

Setup

Geographic Space

- **closed** and **bounded** subset S of 2-dimensional surface, **location** is a point $r \in S$

Land Provision

- $H(r) > 0$ ($H(\cdot)$: exogenous given continuous function)
- $\int_S H(r)dr = 1$

Countries

- C countries, each location belongs to one country
- $S : (S_1, ..., S_C)$

Population

- $\bar{L}$ agents, endowed with one unit of labor (supply inelastically)
- initial distribution $\bar{L}_0(r)$

Time

- **discrete**, $T = 0$ is given

Goods

- continuum, $\omega \in [0, 1]$ (for both production and consumption)

Preferences and Agents' Choices (Consumer Problem)

Period Utility

- agent i resides in r this period t
- lived in a series of locations $\bar{r}_- = (r_0, ..., r_{t-1})$
- $1/[1-\rho]$ is the **C**onstant **E**lasticity of **S**ubstitution with $0 < \rho < 1$

$$u_t^i\left(\bar{r}_-, r\right) = a_t(r)\left[\int_0^1 c_t^{\omega}(r)^{\rho} d\omega\right]^{1/\rho} \varepsilon_t^i(r) \prod_{s=1}^{t} m\left(r_{s-1}, r_s\right)^{-1} \quad (1)$$

Four Parts

- $\mathbf{a_t(r)}$: **amenities** at location r and time t (given for consumers)
- $\mathbf{c_t^{\omega}(r)}$: consumption of good ω at location r and time t
- $\varepsilon_\mathbf{t}^\mathbf{i}(\mathbf{r})$: taste shock (idiosyncratic preferences) distributed to a Fréchet distribution (i.i.d.)
  - constant mean proportional to Ω and variance $\pi^2\Omega^2/6$ with $\Omega < 1$
  - $\Pr[\varepsilon_t^i(r) \leqslant z] = e^{-z^{-1/\Omega}}$ (higher $\Omega \Rightarrow$ greater **heterogeneity**)
- $\mathbf{m(r_{s-1}, r_s)}$: **permanent** **flow-utility** cost of moving from r_{s-1} to r_s

Preferences and Agents' Choices (Cont'd)

Congestion Externalities

$$a_t(r) = \bar{a}(r)\bar{L}_t(r)^{-\lambda} \tag{2}$$

- $\mathbf{a(r) > 0}$: an exogenously given continuous function
- $\bar{L}_t(r)$: population per unit of land at r in period t
- $\lambda \geqslant \mathbf{0}$: fixed parameter, **elasticity** of amenities to population is $-\lambda$

Preferences and Agents' Choices (Cont'd)

Total Income

- $\mathbf{w_t(r)}$: income from work
- $R_t(r)/\bar{L}_t(r)$: income from local ownership of land
  - rents are distributed uniformly across residents
  - alternative assumptions on land ownership can be found in Caliendo et al. (2018)
- no debt contracts ⇒ each period agents simply consume their income

$$u_t^i(\bar{r}_-, r) = \frac{a_t(r)}{\prod_{s=1}^t m(r_{s-1}, r_s)} \frac{w_t(r) + R_t(r)/\bar{L}_t(r)}{P_t(r)} \varepsilon_t^i(r)$$
$$= \frac{a_t(r)}{\prod_{s=1}^t m(r_{s-1}, r_s)} y_t(r) \varepsilon_t^i(r)$$

- $\mathbf{y_t(r)}$: **real income** of an agent in r
- $\mathbf{P_t(r)}$: ideal **price index** at location r in t

$$P_t(r) = \left[\int_0^1 p_t^\omega(r)^{-\rho/(1-\rho)} d\omega\right]^{-(1-\rho)/\rho}$$

Preferences and Agents' Choices (Cont'd)

Assumption 1

Bilateral moving costs can be decomposed into an origin- and a destination-specific component, so $m(s, r) = m_1(s)m_2(r)$. Furthermore, there are no moving costs within a location, so $m(r, r) = 1$ for all $r \in S$.

- **Migration cost**: measured as the percent of **permanent** welfare
- **Enter and leave**: pay the entry migration costs only while being in that country (compensation)
- focus on **net** (rather than gross) migration **flows**
- **Location choice**: depends only on current variables and not on history or future characteristics
- **Discounted total utility**: $\sum_t \beta^t u_t^i(r_{t-}^i, r_t^i)$

Preferences and Agents' Choices (Cont'd)

The **value function** of an agent living at r_0 in period 0, after observing a distribution of taste shocks in all locations, $\bar{\varepsilon}_1^i \equiv \varepsilon_1^i(\cdot)$

$$
\begin{aligned}
V\left(r_0, \bar{\varepsilon}_1^i\right) &= \max_{r_1}\left[\frac{a_1\left(r_1\right)}{m\left(r_0, r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right) + \beta E\left(\frac{V\left(r_1, \bar{\varepsilon}_2^i\right)}{m\left(r_0, r_1\right)}\right)\right] \\
&= \frac{1}{m_1\left(r_0\right)} \max_{r_1}\left[\frac{a_1\left(r_1\right)}{m_2\left(r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right) + \beta E\left(\frac{V\left(r_1, \bar{\varepsilon}_2^i\right)}{m_2\left(r_1\right)}\right)\right] \\
&= \frac{1}{m_1\left(r_0\right)}\left\{\max_{r_1}\left[\frac{a_1\left(r_1\right)}{m_2\left(r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right)\right]\right. \\
&\quad \left. + \beta E\left(\max_{r_2}\left[\frac{a_2\left(r_2\right)}{m_2\left(r_2\right)} y_2\left(r_2\right) \varepsilon_2^i\left(r_2\right) + \frac{V\left(r_2, \bar{\varepsilon}_3^i\right)}{m_2\left(r_2\right)}\right]\right)\right\}
\end{aligned}
$$

- **Red part**: depends only on current variables and taste shocks
  - decision in period 1 is **independent** of history and future ✏️
- **Orange part**: the value of leaving the current location, independent of current location r

Preferences and Agents' Choices (Cont'd)

Period t log utility of an agent ✏️

Using (1) and taking logs,

$$\tilde{u}_t^i(r_0, r_t) = \tilde{u}_t(r_t) - \tilde{m}_1(r_0) - \tilde{m}_2(r_t) + \tilde{\varepsilon}_t^i(r_t)$$

where $\tilde{x} = \ln x$ and $u_t(r)$ denotes the utility level associated with **local amenities** and **real consumption**

$$u_t(r) = a_t(r) \left[\int_0^1 c_t^{\omega}(r)^{\rho} d\omega \right]^{1/\rho} = a_t(r) y_t(r) \tag{3}$$

- desirability of a location, a measure to evaluate social welfare
- not include **mobility costs** and **idiosyncratic preferences**

Preferences and Agents' Choices (Cont'd)

Another measure of social welfare

- **include taste shocks**, ignore mobility costs
- **lack of migration**
  - legal impossibility of moving (lack of info or psychological impediments)
  - tend to overestimate the gains (when evaluating liberalizing restrictions)

Expected period t utility (with reimbursement)

$$E\left(u_t(r)\varepsilon_t^i(r) \mid i \text{ lives in } r\right) = \Gamma(1-\Omega)m_2(r)\left[\int_S u_t(s)^{1/\Omega} m_2(s)^{-1/\Omega} ds\right]^{\Omega} \tag{4}$$

Preferences and Agents' Choices (Cont'd)

Shares of people moving

- density of individuals residing in location s in $t-1$ who prefer location r in period t over all other locations

$$\begin{aligned}\Pr\left(\tilde{u}_t(s,r) \geq \tilde{u}_t(s,v)\forall v \in S\right) &= \frac{\exp\left(\left[\tilde{u}_t(r) - \tilde{m}_2(r)\right]/\Omega\right)}{\int_S \exp\left(\left[\tilde{u}_t(v) - \tilde{m}_2(v)\right]/\Omega\right) dv} \\ &= \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv}\end{aligned} \tag{5}$$

Corresponding to the **fraction of population in** s **that moves to** r:

$$\frac{\ell_t(s,r)}{H(s)\bar{L}_{t-1}(s)} = \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv} \tag{6}$$

- $\ell_{\mathbf{t}}(\mathbf{s},\mathbf{r})$: number of people moving from s to r in t
- $\mathbf{L}_{\mathbf{t-1}}(\mathbf{s})$: total population per unit of land in s at $t-1$

Preferences and Agents' Choices (Cont'd)

Number of people living at r at t must coincide with people who **moved** there or **stayed** there:

$$H(r)\bar{L}_t(r) = \int_S \ell_t(s,r)ds$$

Using (6), this equation can be written as

$$\begin{aligned} H(r)\bar{L}_t(r) &= \int_S \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv} H(s)\bar{L}_{t-1}(s)ds \\ &= \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv} \bar{L} \end{aligned} \tag{7}$$

Technology (Producer Problem)

Production Function (per unit of land)

$$q_t^\omega(r) = \phi_t^\omega(r)^{\gamma_1} z_t^\omega(r) L_t^\omega(r)^\mu$$

- $\phi_{\mathbf{t}}^\omega(\mathbf{r})$: innovation, employ $v\phi_t^\omega(r)^\xi$ **additional units of labor** per unit of land
- $\mathbf{z}_{\mathbf{t}}^\omega(\mathbf{r})$: exogenous local and good-specific productivity shifter
  - random and i.i.d. across good and time periods
  - Fréchet distribution: $F(z,r) = e^{-T_t(r)z^{-\theta}}$ where $T_t(r) = \tau_{\mathbf{t}}(\mathbf{r})\bar{L}(r)^\alpha$
- $\mathbf{L}_{\mathbf{t}}^\omega(\mathbf{r})$: production workers per unit of land at location r at time t

Technology (Producer Problem)

Endogenous dynamic process of $\tau_t(r)$ (given initial productivity $\tau_0(\cdot)$)

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S \eta\tau_{t-1}(s)ds\right]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2} \tag{8}$$

- η: constant such that $\int_S \eta dr = 1$
- $\gamma_2 = 1$ + constant population density: $\mathbb{E}(z_t) = \phi_{t-1}^{\gamma_1}\mathbb{E}(z_{t-1})$ ✏️
  - the distribution of productivity draws is shifted up by past innovations
  - decreasing returns if $\gamma_1 < 1$
- $\gamma_2 < 1$: dynamic evolution of location's technology also depends on aggregate level of technology, $\int_S \eta\tau_t(s)ds$

$\gamma_1, \gamma_2 \in (0, 1)$

- $\gamma_2 = 1 \Rightarrow$ in a BGP, economic activity end up concentrating in a unique point
- $\gamma_1 = \gamma_2 = 0 \Rightarrow$ no incentives to innovate
- **local decreasing returns** + **economywide linear technological progress**

Technology (Producer Problem)

Spatially correlated $z_t^\omega(r)$

- **perfectly correlated** for neighboring locations (distance $\to 0$)
- **independent** (large enough distance)
- $\zeta_t^\omega(r,s)$: correlation in $z_t^\omega(r)$ and $z_t^\omega(s)$
- $\delta(r,s) = d$ denote the distance between r and s
- $\lim_{d\to 0} \zeta_t^\omega(r, s(d)) \to 1$, $\zeta_t^\omega(r, s(d)) = 0$ when δ large enough

One easy example (land divided into regions)

- $\zeta_t^\omega(r,s) = 1$ within a region
- $\zeta_t^\omega(r,s) = 0$ otherwise

Technology (Producer Problem)

Market Structure: perfect local competition

- linear profits in land $\Rightarrow$ small interval = continuum of firms compete in **prices** (**Bertrand**)
- factor prices and transport costs will be **similar** in a small interval
- pricing will be similar locally $\Rightarrow$ **zero profits** (after covering investment $w_t(r)v\phi_t^{\omega}(r)^{\xi}$)
- **Firm:** bid for land $\rightarrow$ win the land auction $\rightarrow$ produce $\rightarrow$ profits always zero
- dynamic innovation decision problem $\Leftrightarrow$ a sequence of **static** innovation decisions (maximize static profits)
- solve only static optimization and disregard (8) (Desmet and Rossi-Hansberg, 2014)

Technology (Producer Problem)

Producer's problem

$$\max_{L_l^\omega(r),\phi_i^\omega(r)} p_t^\omega(r,r)\phi_t^\omega(r)^{\gamma_1} z_t^\omega(r) L_l^\omega(r)^\mu - w_t(r)L_t^\omega(r)$$
$$-w_t(r)\nu\phi_t^\omega(r)^\xi - R_t(r)$$

Two FOCs

$$\mu p_t^\omega(r,r)\phi_t^\omega(r)^{\gamma_1} z_t^\omega(r) L_t^\omega(r)^{\mu-1} = w_t(r) \tag{9}$$

$$\gamma_1 p_t^\omega(r,r)\phi_t^\omega(r)^{\gamma_1-1} z_t^\omega(r) L_t^\omega(r)^{\mu} = \xi w_t(r) v\phi_t^\omega(r)^{\xi-1} \tag{10}$$

Firm's bid rent per unit of land

$$R_t(r) = p_t^\omega(r,r)\phi_t^\omega(r)^{\gamma_1} z_t^\omega(r) L_t^\omega(r)^\mu - w_t(r)L_t^\omega(r) - w_t(r)v\phi_t^\omega(r)^\xi \tag{11}$$

which ensures all firms make **zero profits**.

Technology (Producer Problem)

Using (9) and (10) gives ✏️

$$\frac{L_t^\omega(r)}{\mu} = \frac{\xi v \phi_t^\omega(r)^\xi}{\gamma_1} \tag{12}$$

Total employment = production + innovation

$$\bar{L}_t^\omega(r) = L_t^\omega(r) + v\phi_t^\omega(r)^\xi = \frac{L_t^\omega(r)}{\mu}\left[\mu + \frac{\gamma_1}{\xi}\right] \tag{13}$$

Note also that (in equilibrium $R_t(r)$ is **taken as given** by firms) ✏️

$$R_t(r) = \left[\frac{\xi(1-\mu)}{\gamma_1} - 1\right] w_t(r) v \phi_t^\omega(r)^\xi \tag{14}$$

Lemma 1

The decisions of how much to innovate, $\phi_t^\omega(r)$, and how many workers to hire per unit of land, $\bar{L}_t^\omega(r)$, are ***independent*** *of the local idiosyncratic productivity draws, $z_t^\omega(r)$, and so are identical across goods ω.*

Technology (Producer Problem)

Price of a good produced at r and sold at r ✏️

$$p_t^\omega(r,r) = \left[\frac{1}{\mu}\right]^\mu \left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu} \left[\frac{\gamma_1 R_t(r)}{w_t(r)\nu\left(\xi(1-\mu)-\gamma_1\right)}\right]^{(1-\mu)-(\gamma_1/\xi)} \frac{w_t(r)}{z_t^\omega(r)} \tag{15}$$

Rewrite this as

$$p_t^\omega(r,r) = \frac{mc_t(r)}{z_t^\omega(r)} \tag{16}$$

where $mc_t(r)$ (**given**) denotes the input cost in location r at time t

$$mc_t(r) \equiv \left[\frac{1}{\mu}\right]^\mu \left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu} \left[\frac{\gamma_1 R_t(r)}{w_t(r)\nu\left(\xi(1-\mu)-\gamma_1\right)}\right]^{(1-\mu)-(\gamma_1/\xi)} w_t(r) \tag{17}$$

Eaton and Kortum (2002): price distribution, probability of exporting, share of exports

Prices, Export Probabilities, and Export Shares

iceberg cost of transporting from r to s ($\varsigma(s,r) \geqslant 1$)

$$p_t^\omega(s,r) = p_t^\omega(r,r)\varsigma(s,r) = \frac{mc_t(r)\varsigma(s,r)}{z_t^\omega(r)} \tag{18}$$

Assumption 2

$\varsigma(\cdot,\cdot) : S \times S \to \mathbb{R}$ *is* ***symmetric***.

Probability density (produced in r is bought in s)

$$\pi_t(s,r) = \frac{T_t(r)\left[mc_t(r)\varsigma(r,s)\right]^{-\theta}}{\int_S T_t(u)\left[mc_t(u)\varsigma(u,s)\right]^{-\theta} du} \quad \text{all } r,s \in S \tag{19}$$

Price index

$$P_t(s) = \Gamma\left(\frac{-\rho}{(1-\rho)\theta} + 1\right)^{-(1-\rho)/\rho} \left\{\int_S T_t(u)\left[mc_t(u)\varsigma(s,u)\right]^{-\theta} du\right\}^{-1/\theta} \tag{20}$$

Trade Balance

Total revenue in r ✏️

$$w_t(r)H(r)[L_t(r) + v\phi_t(r)^{\xi}] + H(r)R_t(r) = \frac{1}{\mu}w_t(r)H(r)L_t(r)$$

- location by location
- no mechanism for borrowing from or lending to other agents

Market Clearing (total revenue = total expenditure)

$$w_t(r)H(r)\bar{L}_t(r) = \int_S \pi_t(s,r)w_t(s)H(s)\bar{L}_t(s)ds \quad \text{all } r \in S \tag{21}$$

- Fraction of goods s buys from r, $\pi_t(s,r)$, is **equal** to fraction of expenditure on goods from r (Eaton and Kortum, 2002)
- ω is dropped: number of workers not depend on good
- $\mathbf{L}$ is replaced by $\bar{\mathbf{L}}$: **proportion** of total workers to production workers is constant across r

Equilibrium

Define a dynamic competitive equilibrium

Definition 1

Given a set of locations, S, and their initial technology, amenity, population, and land functions $(\tau_0, \bar{a}, \bar{L}_0, H) : S \to \mathbb{R}_{++}$, as well as their bilateral trade and migration cost functions $\varsigma, m : S \times S \to \mathbb{R}_{++}$.
A competitive equilibrium is a set of functions $(u_t, \bar{L}_t, \phi_t, R_t, w_t, P_t, \tau_t, T_t) : S \to \mathbb{R}_{++}$ for all $t = 1, ...$, as well as a pair of functions $(p_i^{\cdot}, c_i^{\cdot}) : [0, 1] \times S \to \mathbb{R}_{++}$ for all $t = 1, ...$, such that for all $t = 1, ...$:

Equilibrium

1. **Firms** optimize and **markets clear** ((9), (10) and (13) hold at all r).
2. The **share of income** of s spent on goods of r is given by (17) and (19) for all $r, s \in S$.
3. **Trade balance** implies that (21) holds for all $r \in S$.
4. **Land markets** are in equilibrium, so land is assigned to the highest bidder (by (14)). ✏️

$$R_t(r) = \left[\frac{\xi - \mu\xi - \gamma_1}{\mu\xi + \gamma_1}\right] w_t(r)\bar{L}_t(r)$$

5. Given **migration** costs and idiosyncratic preferences, people choose where to live, (7) holds for all $r \in S$.
6. **Utility** associated with real income and amenities in r is given by (20) and ✏️

$$u_t(r) = a_t(r)\frac{w_t(r) + R_t(r)/\bar{L}_t(r)}{P_t(r)} = \bar{a}(r)\bar{L}_t(r)^{-\lambda}\frac{\xi}{\mu\xi + \gamma_1}\frac{w_t(r)}{P_t(r)} \quad \forall r \in S \tag{22}$$

7. **Labor markets** clear: $\int_S H(r)\bar{L}_t(r)dr = \bar{L}$.
8. **Technology** evolves as (8) for all $r \in S$.

Equilibrium

Assumption 3

$\bar{a}(\cdot), H(\cdot), \tau_0(\cdot), \bar{L}_0(\cdot) : S \to \mathbb{R}_{++}$, and $m(\cdot,\cdot), \varsigma(\cdot,\cdot) : S \times S \to \mathbb{R}_{++}$ are continuous functions.

No Discontinuity

- make functions steep at borders (natural geographic barriers)

Discrete approximation

- existence, uniqueness, parameter restrictions (Allen and Arkolakis, 2014)
- for quantification and calibration

Simplification

- manipulate the system of equations
- reduce to **wages**, **employment levels** (labor density), and **utility** in all locations

Equilibrium

Lemma 2

For any t and for all $r \in S$, given $\bar{a}(\cdot)$, $\tau_t(\cdot)$, $\bar{L}_{t-1}(\cdot)$, $\varsigma(\cdot,\cdot)$, $m(\cdot,\cdot)$, and $H(\cdot,\cdot)$, the equilibrium wage, $w_t(\cdot)$, population density $\bar{L}_t(\cdot)$, and utility $u_t(\cdot)$ schedules satisfy equations (7) as well as

$$w_t(r) = \bar{w}\left[\frac{\bar{a}(r)}{u_t(r)}\right]^{-\frac{\theta}{1+2\theta}} \tau_t(r)^{\frac{1}{1+2\theta}} \bar{L}_t(r)^{\frac{\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta}{1+2\theta}} \tag{23}$$

and

$$\begin{aligned}&\left[\frac{\bar{a}(r)}{u_t(r)}\right]^{-\frac{\theta(1+\theta)}{1+2\theta}} \tau_t(r)^{-\frac{\theta}{1+2\theta}} H(r)^{\frac{\theta}{1+2\theta}} \\ &\times \bar{L}_t(r)^{\lambda\theta-\frac{\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\theta\right]\right.} \\ &= \kappa_1 \int_S \left[\frac{\bar{a}(s)}{u_t(s)}\right]^{\frac{\theta^2}{1+2\theta}} \tau_t(s)^{\frac{1+\theta}{1+2\theta}} H(s)^{\frac{\theta}{1+2\theta}} \varsigma(r,s)^{-\theta} \\ &\times \bar{L}_t(s)^{1-\lambda\theta+\frac{1+\theta}{1+2}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]} ds, \text{ where } \kappa_1 \text{ is a constant.}\end{aligned} \tag{24}$$

Equilibrium

Lemma 3

A solution $w_t(\cdot)$, $\bar{L}_t(\cdot)$, and $u_t(\cdot)$ that satisfies (7), (23), and (24) exists and is unique if $\alpha/\theta + \gamma_1/\xi < \lambda + 1 - \mu + \Omega$. Furthermore, the solution can be found with an iterative procedure. (Zabreyko et al., 1975)

Intuition: agglomeration do not dominate congestion forces

- α/θ: local production externalities
- γ_1/ξ: degree of returns to innovation
- λ: negative elasticity of amenities to density
- $1 - \mu$: decreasing returns to local labor
- Ω: variance of taste shocks

Proposition 1

An equilibrium of this economy ***exists*** *and is* ***unique*** *if $\alpha/\theta + \gamma_1/\xi < \lambda + 1 - \mu + \Omega$.*

Outline

The Balanced-Growth Path

Three Cases

- all regions **grow at the same rate**
- eventually concentrate to one point
- cycle without reaching a BGP

The growth rate of $\tau_t(r)$ from (8)

$$\frac{\tau_{t+1}(r)}{\tau_t(r)} = \phi_t(r)^{\theta\gamma_1} \left[\int_S \eta \frac{\tau_t(s)}{\tau_t(r)} ds \right]^{1-\gamma_2}$$

Divide both sides of the equation by the corresponding equation for s, combined with (12) ✏️

$$\frac{\tau_t(s)}{\tau_t(r)} = \left[\frac{\phi(s)}{\phi(r)} \right]^{\frac{\theta\gamma_1}{1-\gamma_2}} = \left[\frac{\bar{L}(s)}{\bar{L}(r)} \right]^{\frac{\theta\gamma_1}{[1-\gamma_2]\xi}}$$

The Balanced-Growth Path

Unique positive solution existence (Zabreyko et al., 1975)

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} + \frac{\gamma_1}{[1-\gamma_2]\xi} \leqslant \lambda + 1 - \mu + \Omega \tag{25}$$

- **New term**: **dynamic agglomeration effect** from local investments in technology as well as diffusion
- $1 - \gamma_2 = 0$: no diffusion, no BGP
- Dispersion forces have to be large enough relative to all agglomeration forces

Lemma 4

If (25) holds, then there exists a ***unique*** *BGP with a* ***constant distribution*** *of employment densities $\bar{L}(\cdot)$ and innovation $\phi(\cdot)$. In the BGP $\tau_t(r)$ grows at a constant rate for all $r \in S$.*

The BGP welfare grows uniformly everywhere at the rate $\frac{u_{t+1}(r)}{u_t(r)} = \left[\frac{\tau_{t+1}(r)}{\tau_t(r)}\right]^{1/\theta}$

The Balanced-Growth Path

The growth rate of world utility/real output (in the BGP)

- a function of population size, the distribution of employment in space

Lemma 5

In a balanced-growth path, under the condition of Lemma 4, aggregate welfare and aggregate real consumption grow according to

$$\frac{u_{t+1}(r)}{u_t(r)} = \left[\frac{\int_0^1 c_{t+1}^{\omega}(r)^{\rho} d\omega}{\int_0^1 c_t^{\omega}(r)^{\rho} d\omega}\right]^{\frac{1}{\rho}} = \eta^{\frac{1-\gamma_2}{\theta}} \left[\frac{\gamma_1/\nu}{\gamma_1 + \mu\xi}\right]^{\frac{\nu_1}{\xi}} \left[\int_S \bar{L}(s)^{\frac{\theta_1}{1-\gamma_{12}(\xi)}} ds\right]^{\frac{1-\gamma\gamma}{\theta}} \tag{26}$$

Strong scale effects

- growth of aggregate consumption would be an **increasing** function of world population
- **Debate**: **no acceleration** in the growth of income per capita in the US in spite of increase in its population (Jones, 1995)
- **Our model**: world economy not in the BGP + population is constant
- **Eliminate** by making the **cost of innovation** an increasing function of population size

Outline

Parameters and Inputs

To compute the equilibrium, we need

- 12 parameters used in the equations above
- initial productivity levels and amenities for all locations
- bilateral migration costs and transport costs between any two locations

Parameter Values

- Assigning parameter values from the existing literature and estimation by real data.

TABLE 1
PARAMETER VALUES

Parameter	Target/Comment
1. Preferences: $\Sigma_t \beta^t u_t(r)$, where $u_t(r) = \bar{a}(r)\overline{L_t}(r)^{-\lambda}(r)[\int_0^1 c_t^{\omega}(r)^{\rho} d\omega]^{1/\rho}$ and $u_0(r) = e^{\psi \tilde{u}(r)}$	
$\beta = .965$	Discount factor
$\rho = .75$	Elasticity of substitution of 4 (Bernard et al. 2003)
$\lambda = .32$	Relation between amenities and population
$\Omega = .5$	Elasticity of migration flows with respect to income (Monte et al. 2018)
$\psi = 1.8$	Deaton and Stone (2013)
2. Technology: $q_t^{\omega}(r) = \phi_t^{\omega}(r)^{\gamma_1} z_t^{\omega}(r) L_t^{\omega}(r)^{\mu}$, $F(z, r) = e^{-T_t^{\omega}(r) z^{-\theta}}$, and $T_t^{\omega}(r) = \tau_t(r)\overline{L_t}(r)^{\alpha}$	
$\alpha = .06$	Static elasticity of productivity to density (Carlino et al. 2007)
$\theta = 6.5$	Trade elasticity (Eaton and Kortum 2002; Simonovska and Waugh 2014)
$\mu = .8$	Labor or nonland share in production (Greenwood et al. 1997; Desmet and Rappaport 2017)
$\gamma_1 = .319$	Relation between population distribution and growth
3. Evolution of productivity: $\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1}[\int_S \eta \tau_{t-1}(s) ds]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2}$ and $\psi(\phi) = \nu \phi^{\xi}$	
$\gamma_2 = .993$	Relation between population distribution and growth
$\xi = 125$	Desmet and Rossi-Hansberg (2015)
$\nu = .15$	Initial world growth rate of real GDP of 2%
4. Trade Costs	
$\varsigma_{\text{rail}} = .1434$	
$\varsigma_{\text{no\_rail}} = .4302$	
$\varsigma_{\text{major\_road}} = .5636$	
$\varsigma_{\text{other\_road}} = 1.1272$	Allen and Arkolakis (2014)
$\varsigma_{\text{no\_road}} = 1.9726$	
$\varsigma_{\text{water}} = .0779$	
$\varsigma_{\text{no\_water}} = .0779$	
$\Upsilon = .393$	Elasticity of trade flows with respect to distance of −.93 (Head and Mayer 2014)

Amenity Parameter: $\lambda = 0.32$

From (2)

$$a_t(r) = \bar{a}(r)\bar{L}_t(r)^{-\lambda} \tag{27}$$

$$\log(a(r)) = \mathbb{E}(\log(\bar{a}(r))) - \lambda \log(\bar{L}(r)) + \varepsilon_a(r) \tag{28}$$

- $\mathbb{E}(\log(\bar{a}(r)))$: the mean of $\log(\bar{a}(r))$ across locations
- $\varepsilon_a(r)$: deviation of $\log(\bar{a}(r))$ from the mean)
  - log-normally distributed across locations

Estimation

- Data: amenities and population for 192 metropolitan statistical areas (MSAs) in the United States Desmet and Rossi-Hansberg (2013))
- Reverse causality
  - Instrument for population: an MSA's exogenous productivity level
  - Productivity that is not due to agglomeration economies

Trade Costs

Location

- location r: $1^\circ \times 1^\circ$ grid cell ($180 \times 360 = 64,800$ grid cells in total)
- trade path $g(r,s)$: a continuous and once-differentiable path to ship a good from r to s

The cost of passing through location r (in logs)

$$\begin{aligned}\log\zeta(r) = {} & \log\zeta_{\text{rail}}\text{rail}(r) + \log\zeta_{\text{no\_rail}}[1-\text{rail}(r)] \\ & + \log\zeta_{\text{major\_road}}\text{major\_road}(r) + \log\zeta_{\text{other\_road}}\text{other\_road}(r) \\ & + \log\zeta_{\text{no\_road}}[1-\text{major\_road}(r)-\text{other\_road}(r)] \\ & + \log\zeta_{\text{water}}\text{water}(r) + \log\zeta_{\text{no\_water}}[1-\text{water}(r)]\end{aligned}$$

- $\text{rail}(r), \text{major\_road}(r), \text{other\_road}(r), \text{water}(r)$: equals 1 if there is a road passing through r and zero otherwise
- $\zeta_{\text{rail}}, \zeta_{\text{no\_rail}}, \zeta_{\text{major\_road}}, \zeta_{\text{other\_road}}, \zeta_{\text{no\_road}}, \zeta_{\text{no\_water}}$: values in Allen and Arkolakis (2014)

Trade Costs

Compute the lowest cost between any two cells $r \neq s$ by Fast Marching Algorithm

$$\zeta(r,s) = \left[\inf_{g(r,s)} \int_{g(r,s)} \zeta(u) du \right]^{\tau}$$

- $\int_{g(r,s)} \zeta(u) du$: the line integral of $\zeta(\cdot)$ along the path $g(r,s)$

Migration Costs

From (7):

$$u_1(r) = H(r)^{\Omega}\bar{L}_1(r)^{\Omega}\bar{L}^{-\Omega}\left[\int_s u_1(v)^{1/\Omega} m_2(v)^{-1/\Omega}\, dv\right]^{\Omega} m_2(r) \tag{29}$$

Plugging this into equation (24) that relates the period 1 population distribution to amenities, land, and period 1 productivity and utility, we get:

$$\begin{aligned}
&\frac{\bar{a}(r)}{\hat{m}_2(r)}\left[\tau_1(r)^{\frac{\theta}{1+\eta}} H(r)^{\frac{\theta(1+\eta)}{1+\eta\theta}}\right]\bar{L}_1(r)^{\lambda\frac{-\theta}{1+\eta}}\left[\alpha - 1 + \left[\lambda + \frac{\gamma_1}{\xi}\right][1-\mu]\right]^{\frac{\theta(1+\eta\theta)}{1+\eta}} \\
&\times \bar{L}_1(r)^{\frac{(1+\theta)\Omega}{1+\eta}} = \kappa_1 \int_s \left[\frac{\bar{a}(s)}{\hat{m}_2(s)}\right]^{\frac{\theta\Omega}{1+\eta}} \tau_1(s)^{1+\frac{\theta\Omega}{1+\eta}} H(s)^{\frac{\theta\Omega}{1+\eta}} s(r,s)^{-\theta} \\
&\times \bar{L}_1(s)^{1-\lambda+\frac{\theta(1+\eta\Omega)}{1+\eta}}\left[\alpha - 1 + \left[\lambda + \frac{\gamma_1}{\xi}\right][1-\mu]\right]^{-\frac{\theta\Omega}{1+\eta}} ds
\end{aligned} \tag{30}$$

$$\hat{m}_2(r) = \frac{m_2(r)}{\bar{L}^{\Omega}\left[\int_s u_1(v)^{1/\Omega} m_2(v)^{-1/\Omega}\, dv\right]^{\Omega}} \tag{31}$$

Migration Costs

Lemma 6

The solution to (30), $\hat{m}_2(\cdot)$, exists, is unique, and can be found by iteration ω

- The solution to (30), yields a $\hat{m}_2(\cdot)$ for all $r \in S$
- The values of $m_2(\cdot)$ are identified only up to a scale by (31)

To solve for $m_2(\cdot)$, we need to know:

- productivity in period 1 ($\tau_1(\cdot)$): by productivity evolution (8) from $\tau_0(\cdot)$ and $\bar{L}_0(\cdot)$
- population levels in period 1 ($\bar{L}_1(\cdot)$): data on the population distribution in 2005

Thank You!

Outline

About the Researchers

- Klaus Desmet
  - His research focuses on regional economics, climate change, and political economy.

- Dávid Krisztián Nagy
  - His research focuses on international trade, economic geography and economic growth.

- Esteban Rossi-Hansberg
  - His research specializes in international trade, regional and urban economics, as well as growth and organizational economics.

Read more about this paper in The Economist (in English) or the WeChat tweets 1 and 2 (in Chinese).

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